Introduction to atmospheric inversions with examples of applications in Africa

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Flux Measurements Winter School, Vuwani, South Africa

Overview

- Bayesian inversion modelling approach
- Prior information: CO_2, CH_4, and N_2O
- Sensitivity matrix
- Background concentration
- Observation errors
- Estimates of CO_2 fluxes for Cape Town
- Optimal network for atmospheric monitoring in Africa

So you've got some atmospheric measurements … now what?

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	DATE	TIME	FRAC_DAYS_SINC	FRAC_HRS_SINCE	JULIAN_DAYS	EPOCH_TIME	ALARM	INST_ST	CavityPressure	CavityTemp	DasTemp	EtalonTemp	WarmBoxTemp	species	MPVPosition	OutletValve	solenoid_valves	CO2	CO2_dry	CH4	CH4_dry	H2O
2	07/05/2018	19:07.9	126.3882854	3033.31885	127.3882854	1525677548	0	963	1.40E+02	4.47E+01	2.40E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.68E+02	4.79E+02	1.85E+00	1.87E+00	6.54E
3	07/05/2018	19:08.9	126.388297	3033.319127	127.388297	1525677549	0	963	1.40E+02	4.47E+01	2.40E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.68E+02	4.73E+02	1.85E+00	1.86E+00	6.45E
4	07/05/2018	19:11.4	126.3883266	3033.319839	127.3883266	1525677551	0	963	1.40E+02	4.47E+01	2.40E+01	4.48E+01	4.49E+01	2.00E+00	0.00E+00	2.64E+04	0.00E+00	4.68E+02	4.73E+02	1.85E+00	1.86E+00	6.45E
5	07/05/2018	19:12.9	126.3883433	3033.320239	127.3883433	1525677553	0	963	1.40E+02	4.47E+01	2.39E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.61E+02	4.73E+02	1.85E+00	1.86E+00	6.45E
6	07/05/2018	19:13.9	126.3883556	3033.320534	127.3883556	1525677554	0	963	1.40E+02	4.47E+01	2.39E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.61E+02	4.66E+02	1.85E+00	1.86E+00	6.44E
7	07/05/2018	19:16.2	126.388382	3033.321169	127.388382	1525677556	0	963	1.40E+02	4.47E+01	2.40E+01	4.48E+01	4.49E+01	2.00E+00	0.00E+00	2.64E+04	0.00E+00	4.61E+02	4.66E+02	1.85E+00	1.86E+00	6.44E
8	07/05/2018	19:17.8	126.3884004	3033.32161	127.3884004	1525677558	0	963	1.40E+02	4.47E+01	2.40E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.63E+02	4.66E+02	1.85E+00	1.86E+00	6.44E
9	07/05/2018	19:18.8	126.3884116	3033.321877	127.3884116	1525677559	0	963	1.40E+02	4.47E+01	2.39E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.63E+02	4.68E+02	1.85E+00	1.86E+00	6.58E
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11	07/05/2018	19:22.7	126.3884572	3033.322973	127.3884572	1525677563	0	963	1.40E+02	4.48E+01	2.40E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.74E+02	4.68E+02	1.85E+00	1.86E+00	6.58E
12	07/05/2018	19:23.6	126.3884679	3033.32323	127.3884679	1525677564	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.74E+02	4.79E+02	1.85E+00	1.86E+00	6.45E
13	07/05/2018	19:26.1	126.3884967	3033.32392	127.3884967	1525677566	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	2.00E+00	0.00E+00	2.64E+04	0.00E+00	4.74E+02	4.79E+02	1.85E+00	1.86E+00	6.45E
14	07/05/2018	19:27.6	126.3885135	3033.324324	127.3885135	1525677568	0	963	1.40E+02	4.48E+01	2.40E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.77E+02	4.79E+02	1.85E+00	1.86E+00	6.45E
15	07/05/2018	19:28.6	126.3885251	3033.324602	127.3885251	1525677569	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.77E+02	4.82E+02	1.85E+00	1.86E+00	6.43E
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17	07/05/2018	19:32.5	126.3885704	3033.325691	127.3885704	1525677572	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.73E+02	4.82E+02	1.85E+00	1.86E+00	6.43E
18	07/05/2018	19:33.5	126.3885818	3033.325964	127.3885818	1525677573	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.73E+02	4.77E+02	1.85E+00	1.86E+00	6.42E
19	07/05/2018	19:35.9	126.3886101	3033.326641	127.3886101	1525677576	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	2.00E+00	0.00E+00	2.64E+04	0.00E+00	4.73E+02	4.77E+02	1.85E+00	1.86E+00	6.42E
20	07/05/2018	19:37.5	126.3886287	3033.327088	127.3886287	1525677578	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.77E+02	4.77E+02	1.85E+00	1.86E+00	6.42E
21	07/05/2018	19:38.5	126.3886398	3033.327354	127.3886398	1525677578	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.77E+02	4.82E+02	1.85E+00	1.86E+00	6.44E
22	07/05/2018	19:40.8	126.3886663	3033.327992	127.3886663	1525677581	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	2.00E+00	0.00E+00	2.64E+04	0.00E+00	4.77E+02	4.82E+02	1.85E+00	1.86E+00	6.44E
23	07/05/2018	19:42.4	126.3886848	3033.328435	127.3886848	1525677582	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.81E+02	4.82E+02	1.85E+00	1.86E+00	6.44E
24	07/05/2018	19:43.3	126.3886958	3033.328699	127.3886958	1525677583	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.81E+02	4.86E+02	1.85E+00	1.87E+00	6.50E
25	07/05/2018	19:45.7	126.3887233	3033.329358	127.3887233	1525677586	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	2.00E+00	0.00E+00	2.64E+04	0.00E+00	4.81E+02	4.86E+02	1.85E+00	1.87E+00	6.50E
26	07/05/2018	19:47.1	126.3887397	3033.329752	127.3887397	1525677587	0	963	1.40E+02	4.48E+01	2.39E+01	4.48E+01	4.49E+01	1.00E+00	0.00E+00	2.64E+04	0.00E+00	4.79E+02	4.86E+02	1.85E+00	1.87E+00	6.50E
27	07/05/2018	19:48.1	126.3887509	3033.330023	127.3887509	1525677588	0	963	1.40E+02	4.48E+01	2.40E+01	4.48E+01	4.49E+01	3.00E+00	0.00E+00	2.64E+04	0.00E+00	4.79E+02	4.84E+02	1.85E+00	1.86E+00	6.40E

Bayesian inverse modelling

- Statistical method which calculates fluxes from CO_2 concentrations, high resolution meteorology and atmospheric transport.
- Regularizes the problem by incorporating prior information about the flux components.
- Relies on high precision measurements of CO_2.
- Requires information about boundary concentrations when performing a mesoscale inversion

Bayesian solution

Linear relationship between concentrations and fluxes:

$$\mathbf{c}_{mod} = \mathbf{Hs}$$

Bayesian cost function:

$$J(\mathbf{s}) = \frac{1}{2}((\mathbf{c}_{mod} - \mathbf{c})^T \mathbf{C}_{\mathbf{c}}^{-1} (\mathbf{c}_{mod} - \mathbf{c}) + (\mathbf{s} - \mathbf{s_0})^T \boldsymbol{C}_{\mathbf{s_0}}^{-1} (\mathbf{s} - \mathbf{s_0})$$

Posterior flux estimates:

$$\hat{\mathbf{s}}_{BLS} = \mathbf{s}_0 + \mathbf{C}_{\mathrm{s}_0} \mathbf{H}^T \left(\mathbf{H}\mathbf{C}_{\mathrm{s}_0}\mathbf{H}^T + \mathbf{C}_{\mathbf{c}}\right)^{-\mathbf{1}} (\mathbf{c} - \mathbf{H}\mathbf{C}_{\mathrm{s}_0})$$

Posterior covariance matrix:

$$\mathbf{C}_{\mathbf{s}} = (\mathbf{H}\mathbf{C}_{\mathbf{c}}^{-\mathbf{1}}\mathbf{H} + \mathbf{C}_{\mathrm{s}_0})^{-\mathbf{1}}$$

$$= \mathbf{C}_{\mathrm{s}_0} - \mathbf{C}_{\mathrm{s}_0} \mathbf{H}^T \left(\mathbf{H}\mathbf{C}_{\mathrm{s}_0}\mathbf{H}^T + \mathbf{C}_{\mathbf{c}}\right)^{-\mathbf{1}} \mathbf{H}\mathbf{C}_{\mathrm{s}_0}$$

Tarantola, 2005

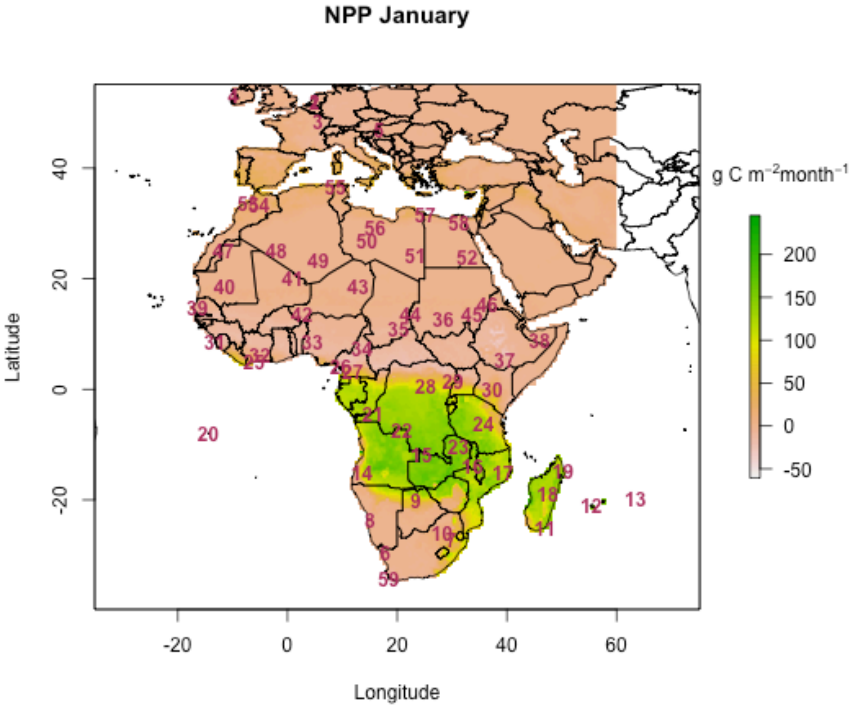
Bayesian Inversion

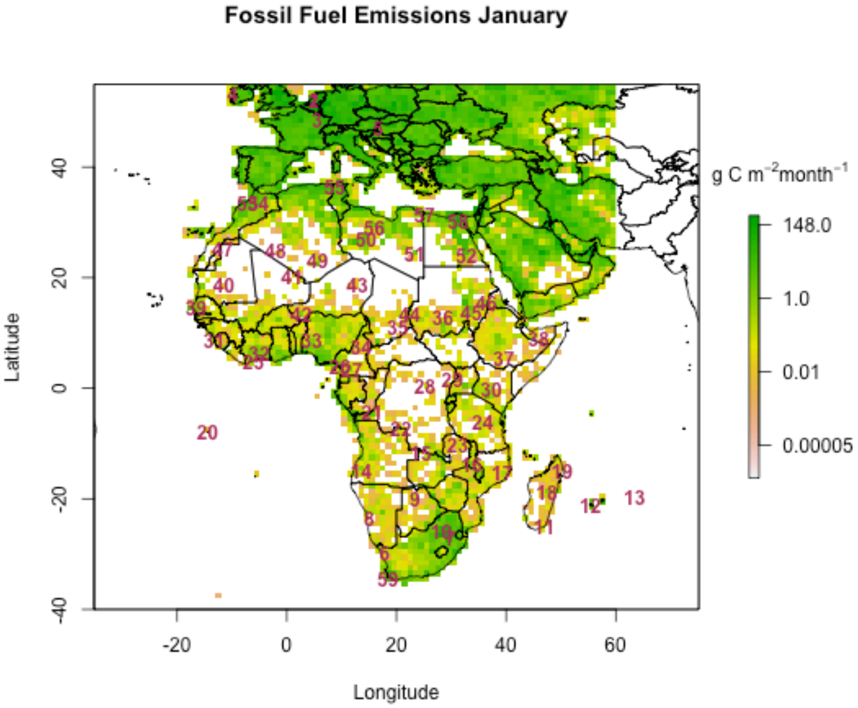
- Typically assumes that the flux uncertainties are Gaussian with covariance matrix $\mathbf{C}_{s_0}$
- and that the observation errors are Gaussian with covariance matrix $\mathbf{C}_\mathbf{c}$
- But this can be a problem when fluxes are only positive or when the error distributions have long tails.

Prior Information

- Need to start off with good initial estimates of the fluxes in your domain.
- The inversion then corrects these fluxes according to the observations.
- The extent to which the inversion can correct the flux depends on
  - The uncertainty assigned to the fluxes (the smaller the uncertainty the smaller the change the inversion can make)
  - How well the observation network views the domain
  - How uncertain the modelled observations are (the greater the uncertainty the smaller the constraint of the observations)

CO_2 Priors


NPP January
Latitude
Longitude
g C m-2month-1
200
150
100
50
0
-50


Fossil Fuel Emissions January
Latitude
Longitude
g C m-2month-1
148.0
1.0
0.01
0.00005

CO_2 Priors

- Need to consider both biogenic fluxes and fossil fuel emissions.
- Make sure to carefully consider all the sources.
- Getting the spatial allocation of fluxes more or less correct, at least relatively speaking, makes a big difference to the inversion results.
- Fossil fuel fluxes can be specified as fixed if these are well constrained in the domain. Although this is rarely the case.
- Fossil fuel and natural fluxes can be solved separately.
- NEE from models has large errors associated with it.

CO_2 Priors

- You can solve for a single flux in each biome (which can cover several pixels) or solve for each pixel.
- You can also solve for a scale factor instead of the fluxes.
- The spatial scale of these fluxes largely determines the computational cost of the inversion.
- The spatial and temporal scale of the fluxes determines the control vector or basis function – $\boldsymbol{s_0}$.
- Although you might start off with lat / lon grids of spatial fluxes, these get collapsed into a vector. Make sure you know how to turn the vector back into you lat / long grid.

CO_2 flux uncertainties

- Fossil fuel flux uncertainties are dependent on how well the anthropogenic activities are quantified. Normally, these emissions are always positive.

$$s_{ff} = AE$$

- Uncertainties can be quantified using error propagation techniques

$$C_{s_{ff}} = \left|s_{ff}\right|^2 \left((\frac{\delta A}{A})^2 \; (\frac{\delta E}{E})^2 \right)$$

Where δA is the uncertainty in the activity data and δE is the uncertainty in the emission factor.

- Another approach is to assign a percentage uncertainty. This is usually the approach when using a global product like EDGAR or ODIAC, which is based on population statistics and National Inventory reporting.

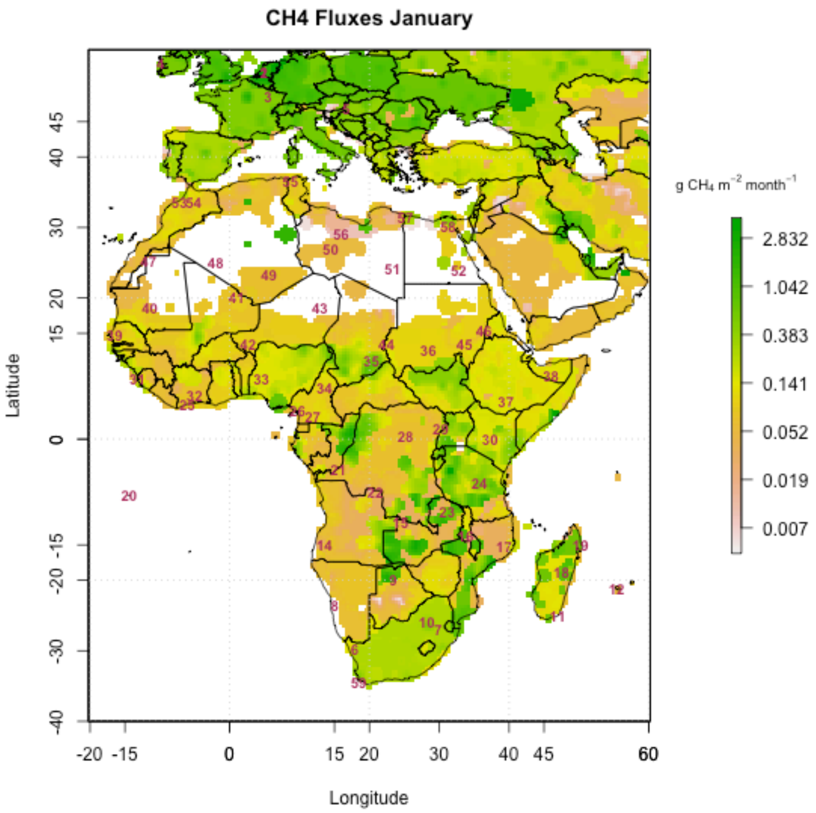
CO_2 flux uncertainties

- Biogenic fluxes can be derived from from a land surface exchange models or digital global vegetation models (DGVMs).
- These models usually give estimates of net ecosystem exchange (NEE), ecosystem respiration (Re), and gross primary production (GPP).

$$NEE = Re + GPP$$

- NEE can be positive or negative depending on which dominates.
- Assigning percentage based uncertainties to NEE usually leads to uncertainties that are too small. So should rather base uncertainty on the GPP component.

CH_4 Priors



- Need to account for CH_4 fluxes from:
  - Release from animals
  - Coal mining
  - Pipeline leakage from natural gas
  - Venting of natural gas at wells
  - Release from landfills
  - Soil absorption
  - Termites
  - Swamps
  - Wetlands

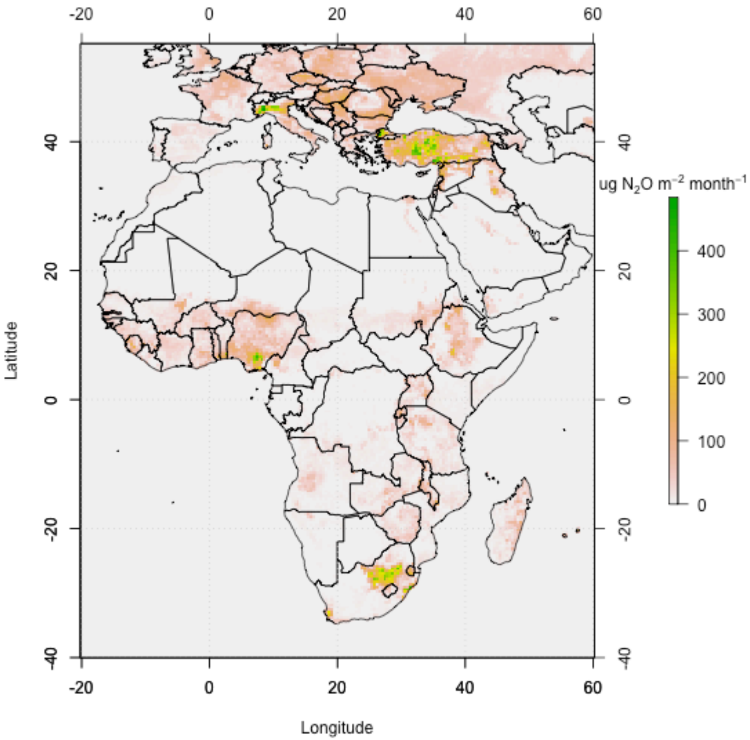
CH_4 Priors

- CH_4 fluxes are mainly positive. Therefore flux uncertainties are usually assumed to be log-normally distributed or to have a truncated Gaussian distribution.
- The solutions for the posterior fluxes and uncertainties need to be adapted to accommodate the change in assumptions.
- If using a truncated Gaussian, any negative posterior fluxes are made to equal zero.
- Log-normal distributions usually require iterative methods to solve for the fluxes.
- Global inversions or inversions of long periods need to account of loss of methane in the atmosphere due to OH.

CH_4 flux uncertainties

- These are usually made proportional to the prior flux.
- Often set at 100% of the prior flux.
- One approach has set the uncertainty of a pixel's flux to the maximum of that pixel and surrounding pixels

N_2O Priors



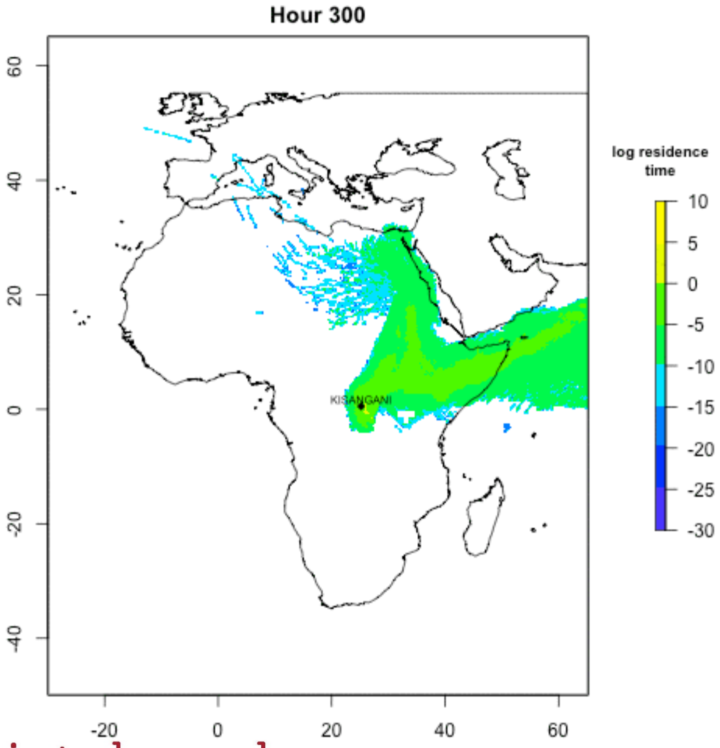
N_2O emissions from

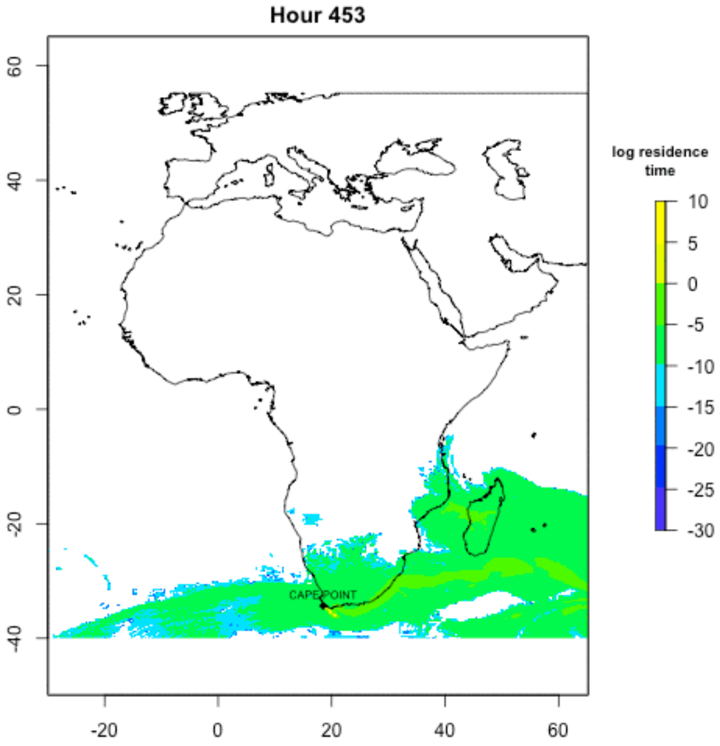
- Agricultural soils
- Fuel combustion
- Animal waste management
- Natural soil emissions
- Oceanic fluxes (which can sometimes be negative)

N_2O Priors

- N_2O fluxes usually positive.
- Uncertainties usually set to be proportional to prior N_2O flux estimates

Sensitivity Matrix





Sensitivity Matrix

s has length m and **c** has length n

	s_1	s_2	s_3	…		s_m
c_1	H_{11}	H_{12}	H_{13}	…		H_{1m}
c_2	H_{21}	H_{22}	H_{23}	…		H_{2m}
c_3	H_{31}	H_{32}	H_{33}	…		H_{3m}
…						
c_n	H_{n1}	H_{n2}	H_{n3}	…		H_{nm}

Sensitivity Matrix

- Lagrangian particle dispersion model run in backward mode can provide the sensitivity of an observation to the sources and the boundary.
- Example models are STILT, FLEXPART and NAME.
- These models are driven by meteorological data from regional climate model.

Background Influence

- In a regional inversion the observed concentration depends on the fluxes within the domain and the concentration at the boundary of the domain.
- These can be provided by a background site or modelled using a chemical transport model, e.g. using MOZART.

Observation Errors

- Measurement error (should be as small as possible)
- Transport model error
- Representation error
- Aggregation error

Estimates of CO_2 fluxes from the city of Cape Town through Bayesian inverse modelling

Alecia Nickless

Supervisors:

Prof. Peter Rayner – University of Melbourne

Prof. Bob Scholes – University of the Witwatersrand

Dr. Birgit Erni – University of Cape Town

Prof. Les Underhill – University of Cape Town

Other contributors:

Ernst-Günther Brunke – SAWS, Dr. Francois Engelbrecht - CSIR


Robben Island
CBD
Cape Point
Hangklip

Why Robben Island and Hangklip?

- Place instruments around Cape Town City, in such a way to constrain estimate fluxes over the peninsula.

- Lighthouses provide ideal housing for instruments, with access to power, and the infrastructure to allow the inlet tube to be positioned at an adequate height.

Picarro Setup at Robben Island and Hangklip

- Each site is equipped with a Picarro Cavity Ring-Down Spectroscopy Analyser (CRDS G2301) for measuring CO2, CH4 and H2O concentrations.

- Instruments were monitored via a 3G internet connection and visited approximately every two months.

- A rotating calibration standard was measured at all three sites, as well as a fully automatic calibration system for each individual site.

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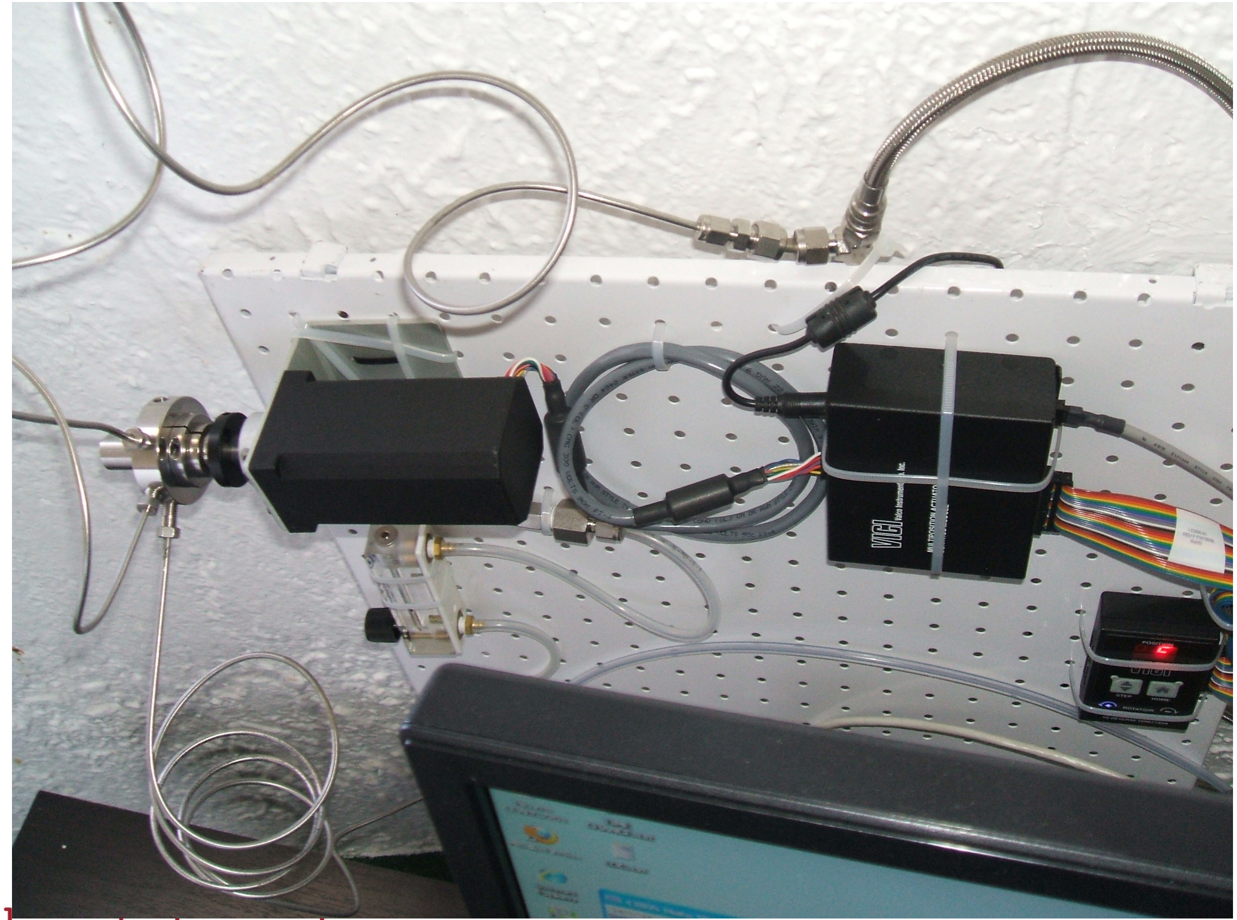
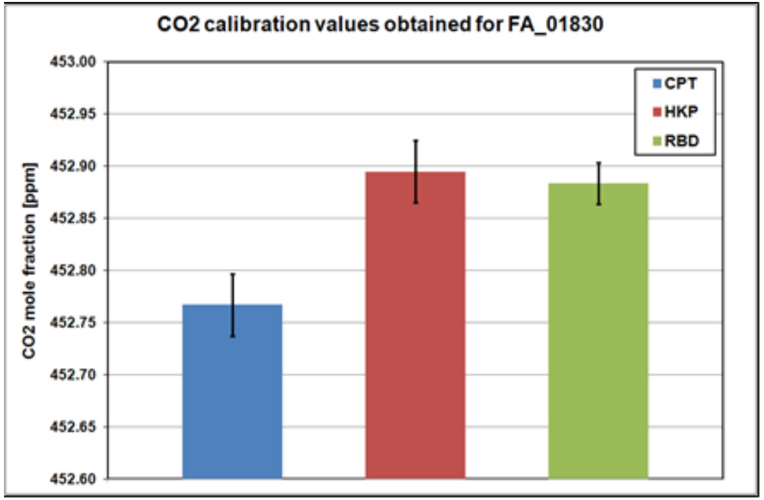


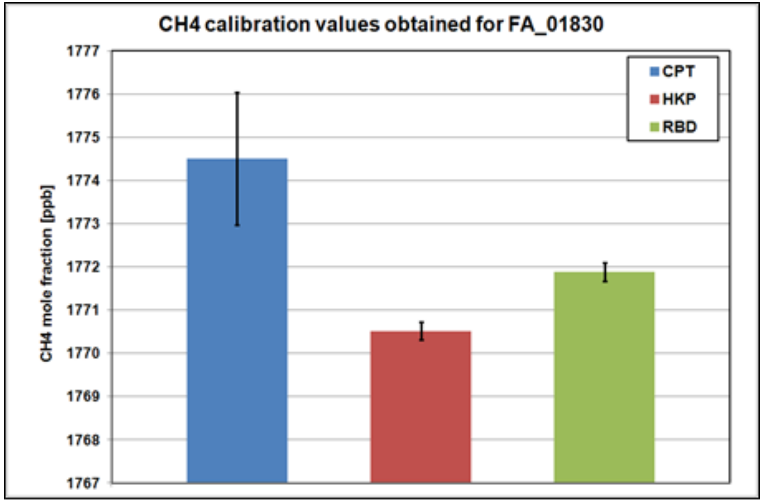
- Calibration system design with input from Dr. Martin Steinbacher of Empa and Ernst Brunke of SAWS.

Inter-calibration between three sites via travelling standard, FA_01830



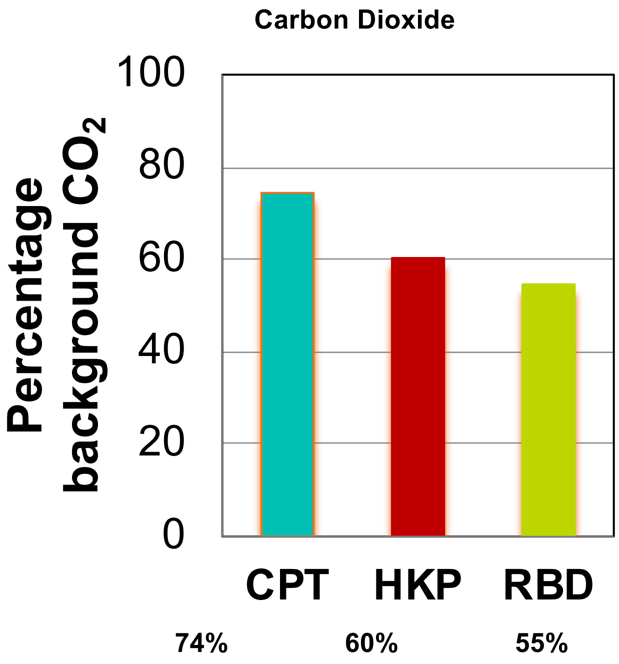
CO_2 difference HKP – CPT: <u>0.13 ppm</u>

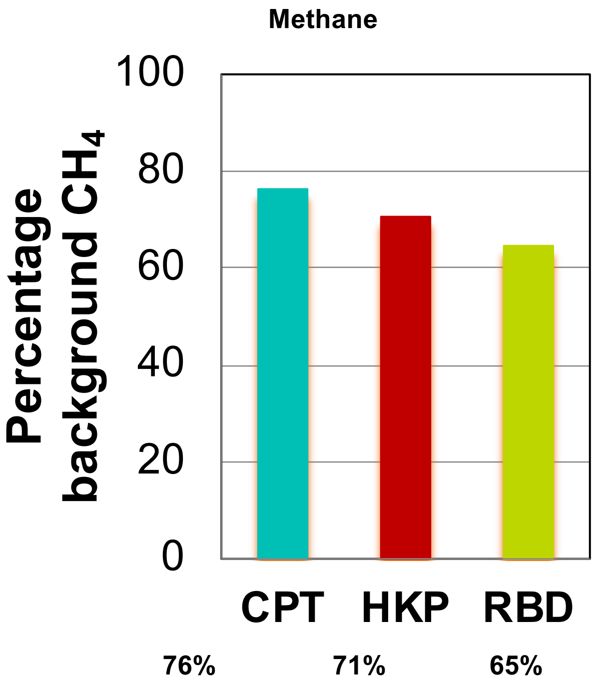
CH_4 difference HKP – CPT : <u>-4 ppb</u>



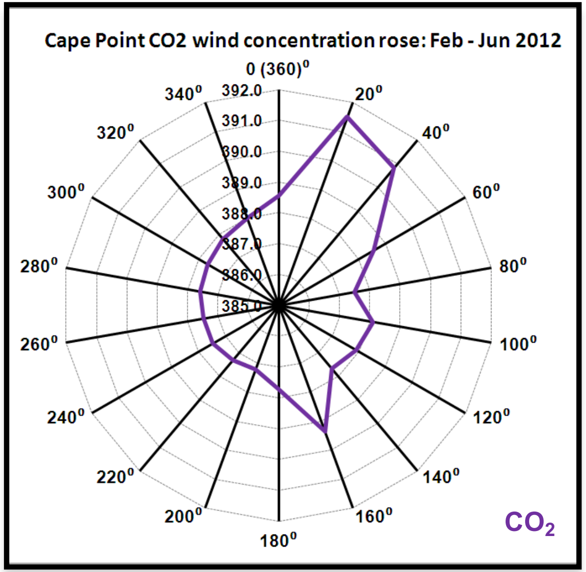
- CPT maintains 10 NOAA lab standards re-analysed every three years at the CCL, Boulder.
- CPT working standards (running once/week) are linked to these NOAA lab standards.
- Inter-comparability differences between sites are significantly less than real differences observed.

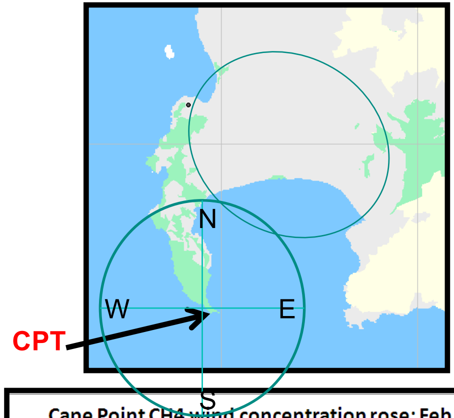
Percentage background CO_2 and CH_4 relative to total data at three sites

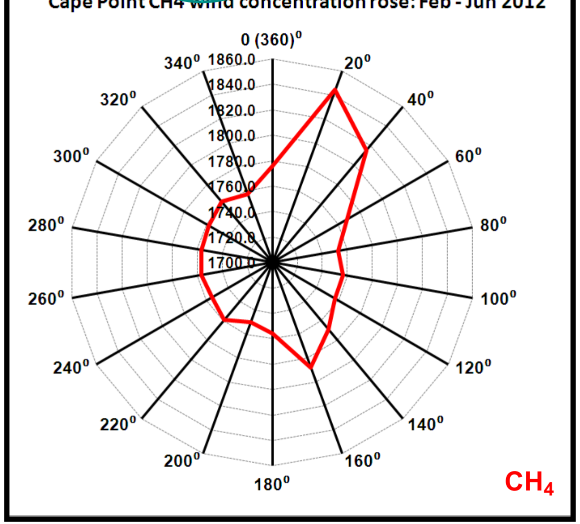




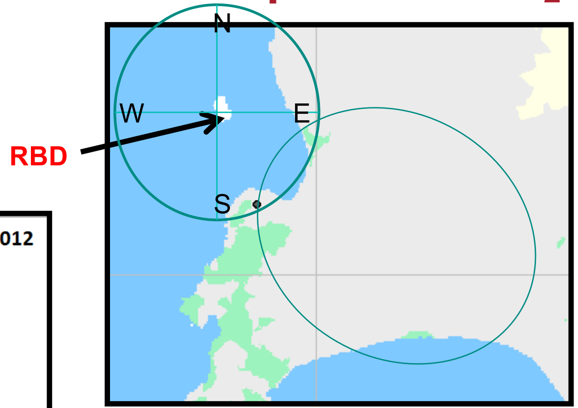
Cape Point angular distribution plots for CO_2 and CH_4

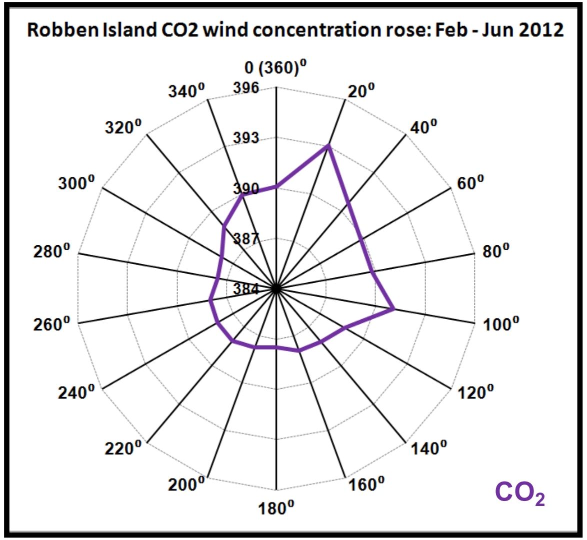


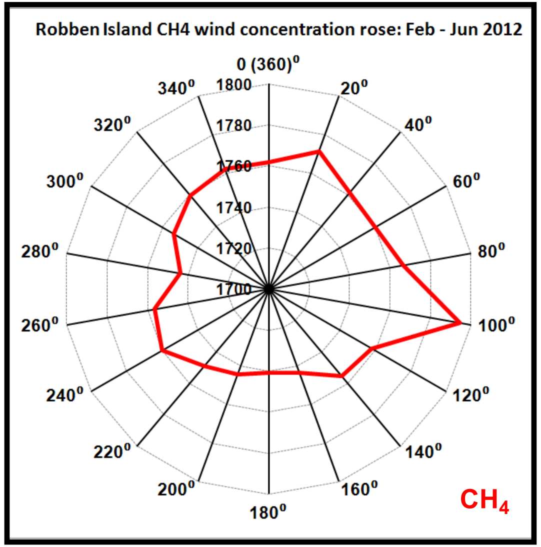




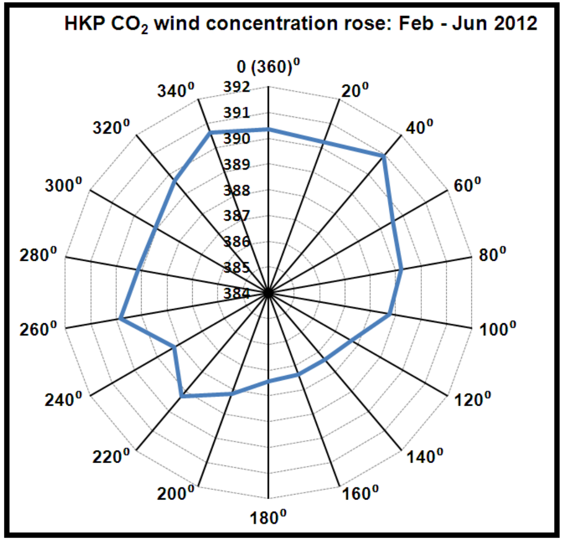
Robben Island angular distribution plots for CO_2 and CH_4

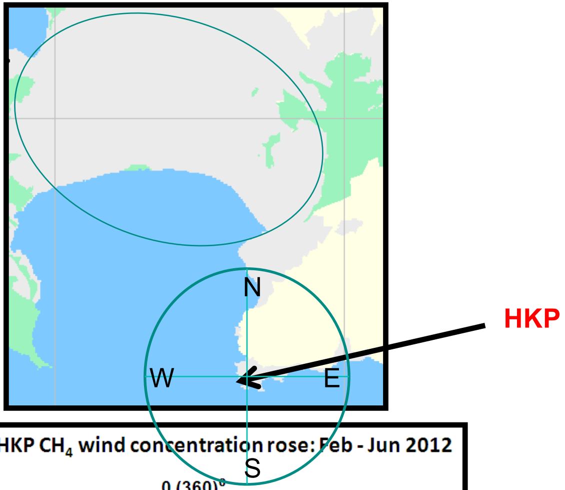


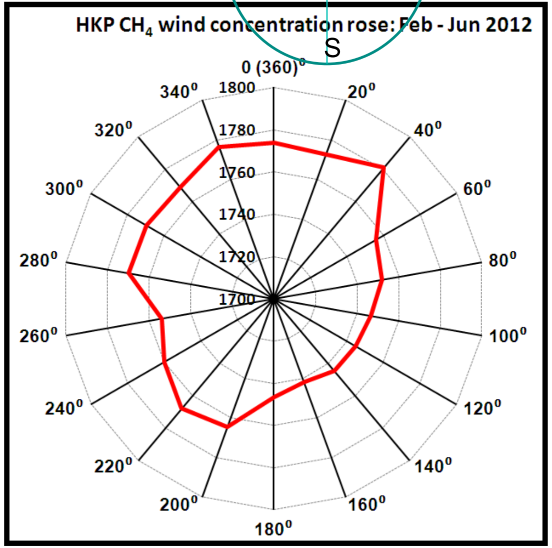




Hangklip angular distribution plots for CO_2 and CH_4







Wind data (courtesy K. Moir) from Rooi Els; 10 km S of HKP.

Case histories using isentropic back-trajectories (Hysplit Model) from NOAA-ESRL

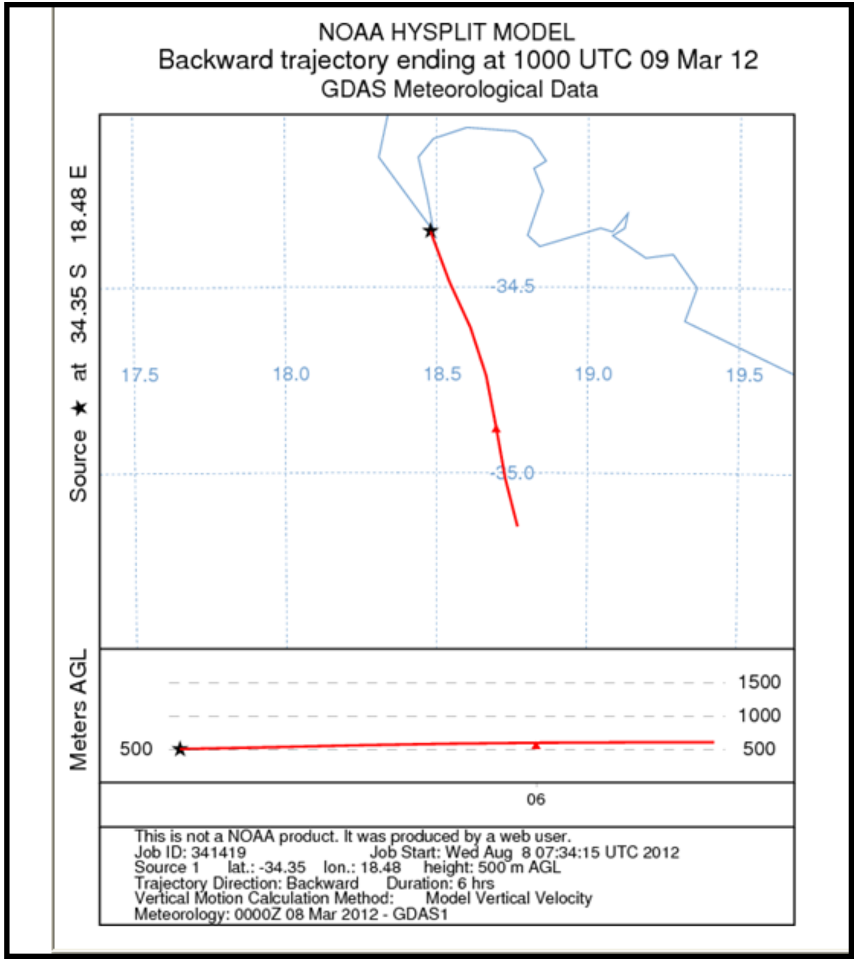
- Trajectories are selected for eight major directions showing CO_2 and CH_4 median values (several cases) at trajectory arrivals for the 3 sites.

- Certain instances give rise to similar air masses reaching 2 or all 3 sites thereby confirming comparable GHG concentrations.

- Alternatively, cases arise where <u>background</u> air flows over one site and then over the Cape Town <u>urban area</u> before reaching another site.

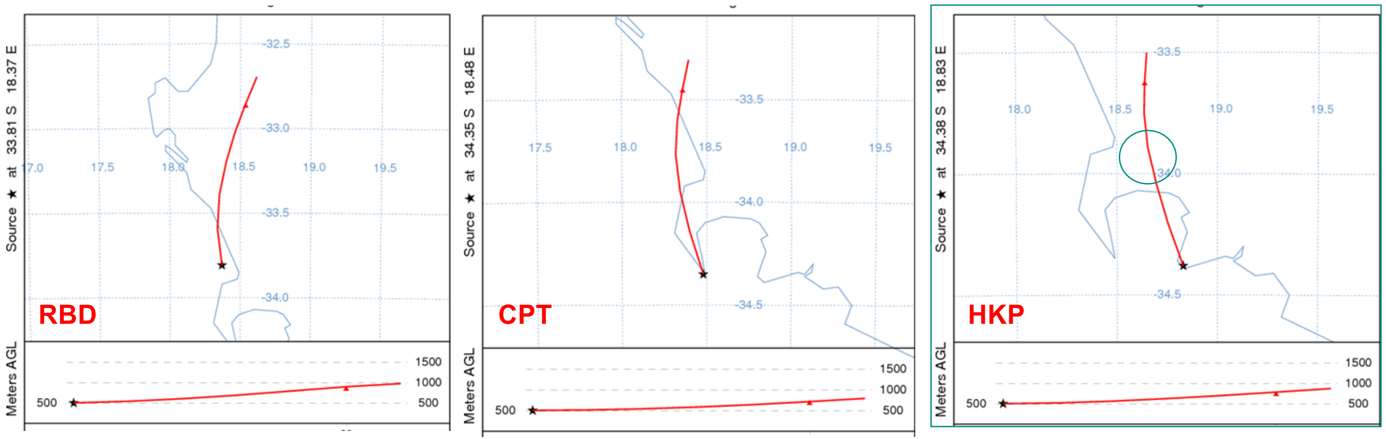
- The latter provides an opportunity to assess GHG emissions from the greater Cape Town region.

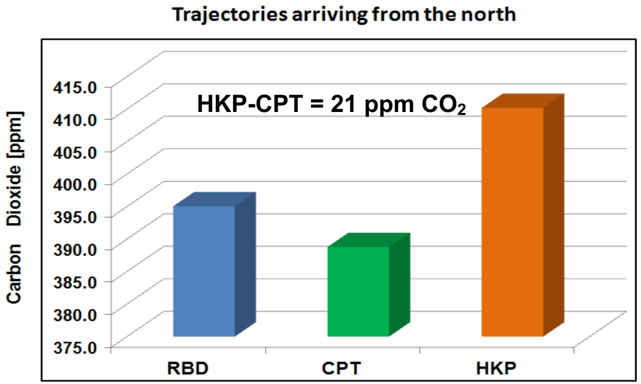
Six hour back trajectory (Hysplit Model - NOAA-ESRL)

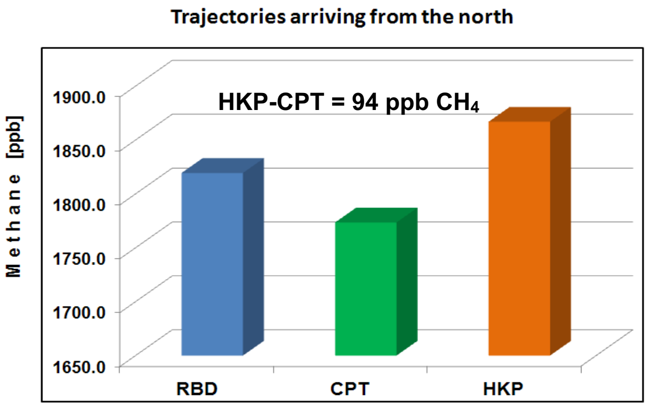


Source: http://www.esrl.noaa.gov/gmd

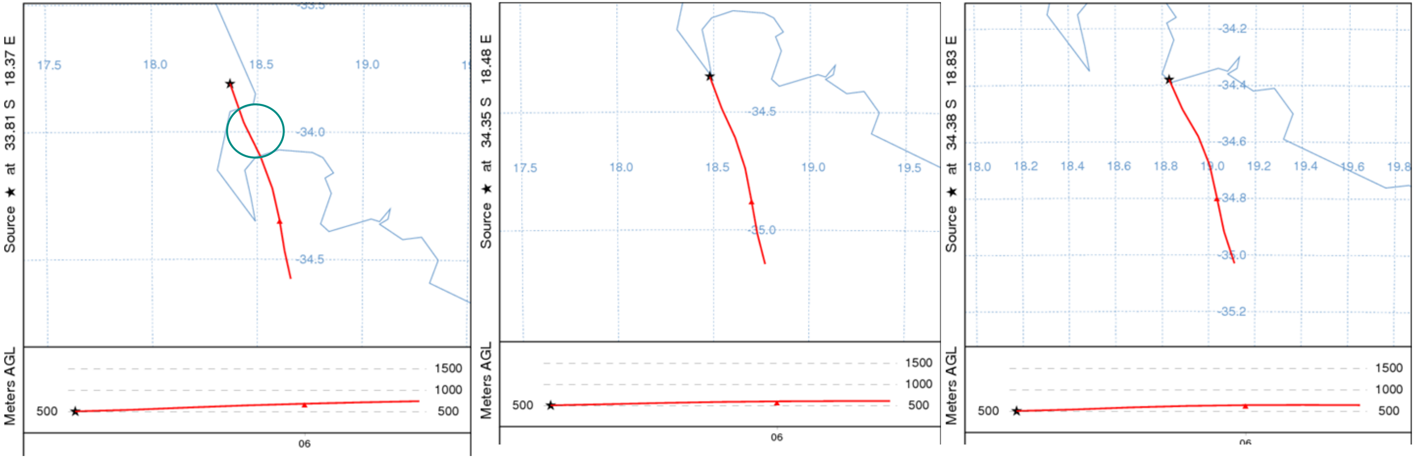
Trajectories from the north

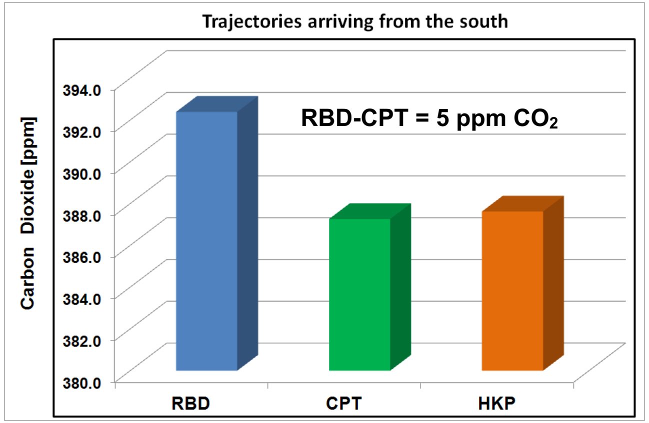


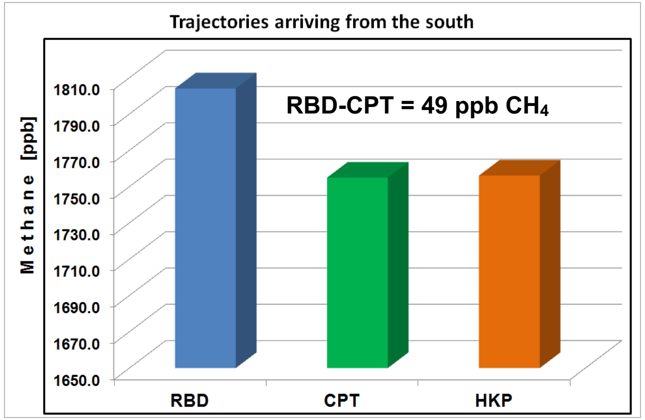




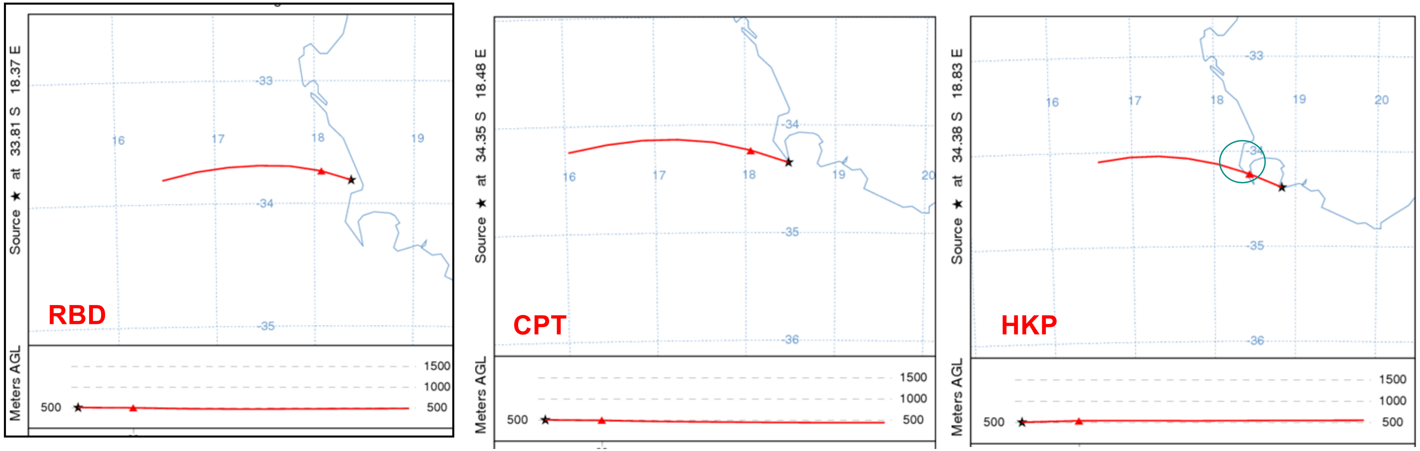
Trajectories from the south

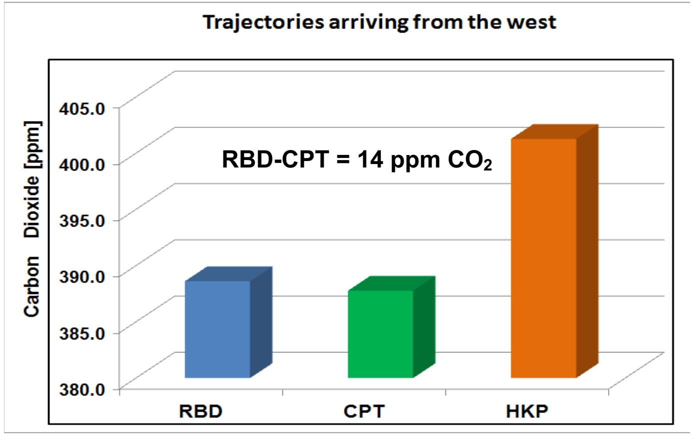


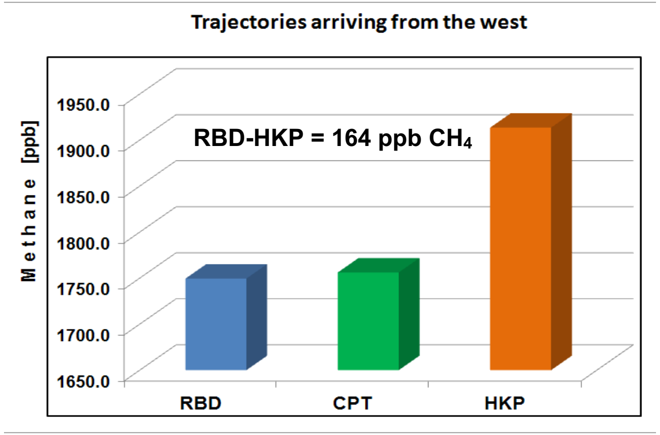




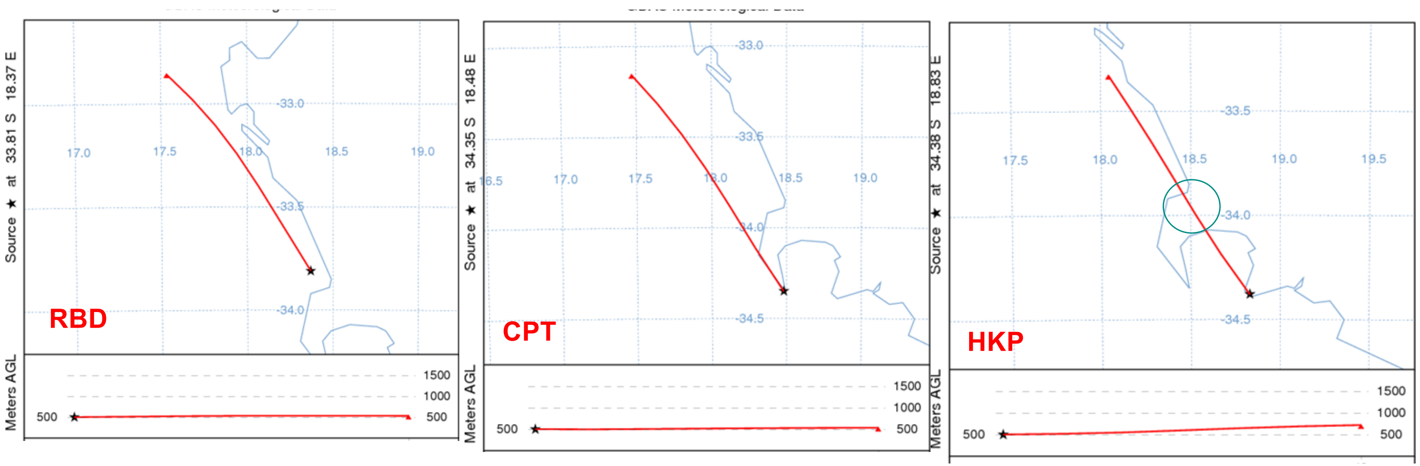
Trajectories from the west

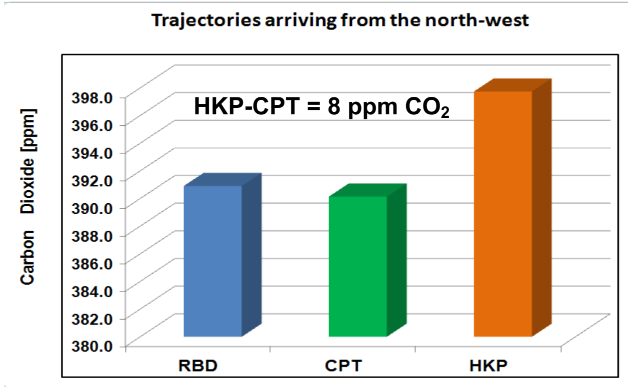


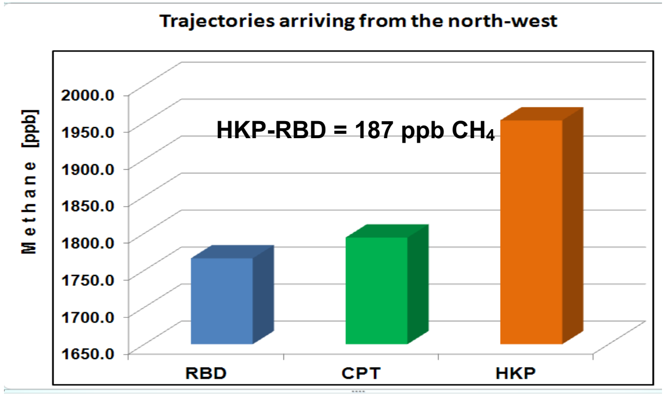


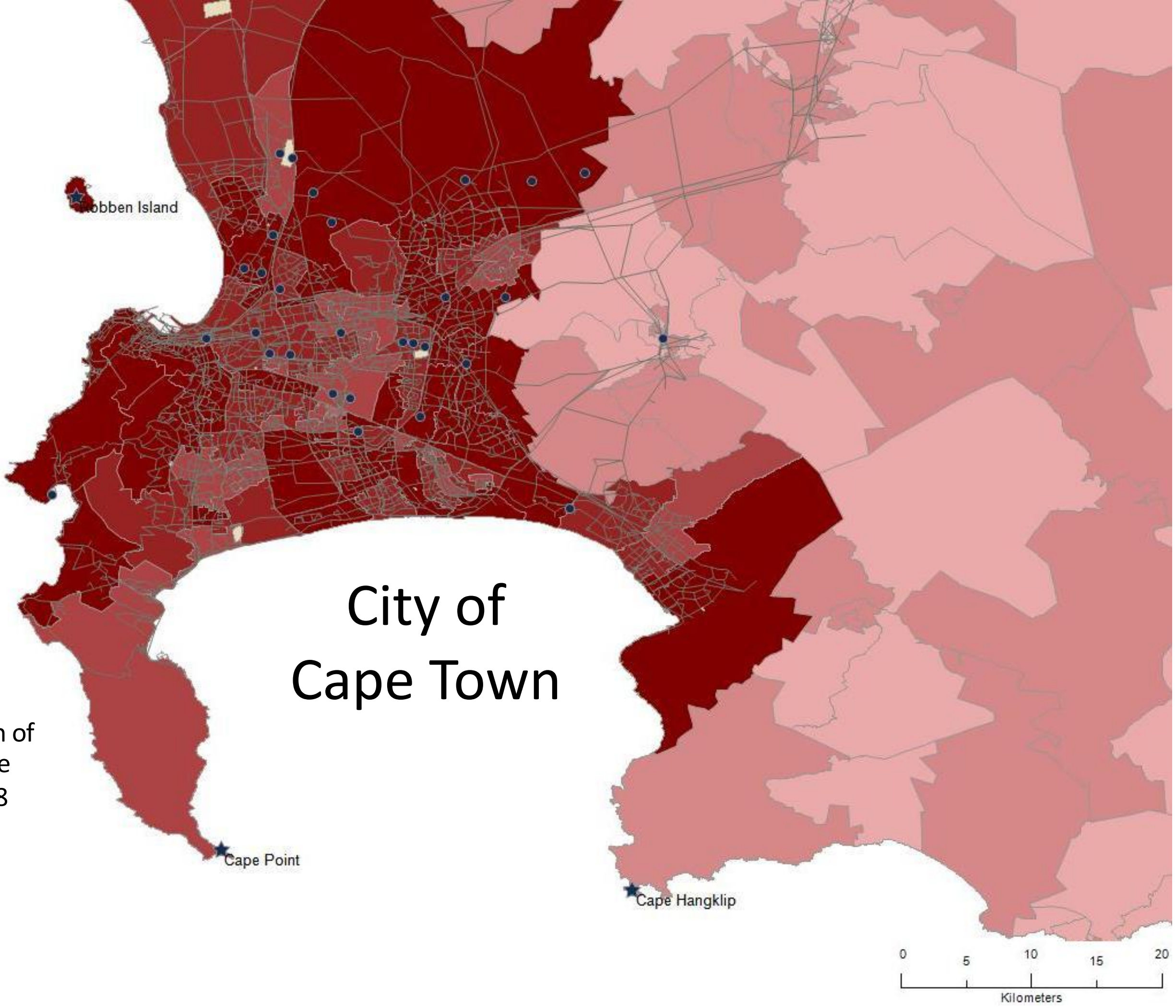


Trajectories from the north-west

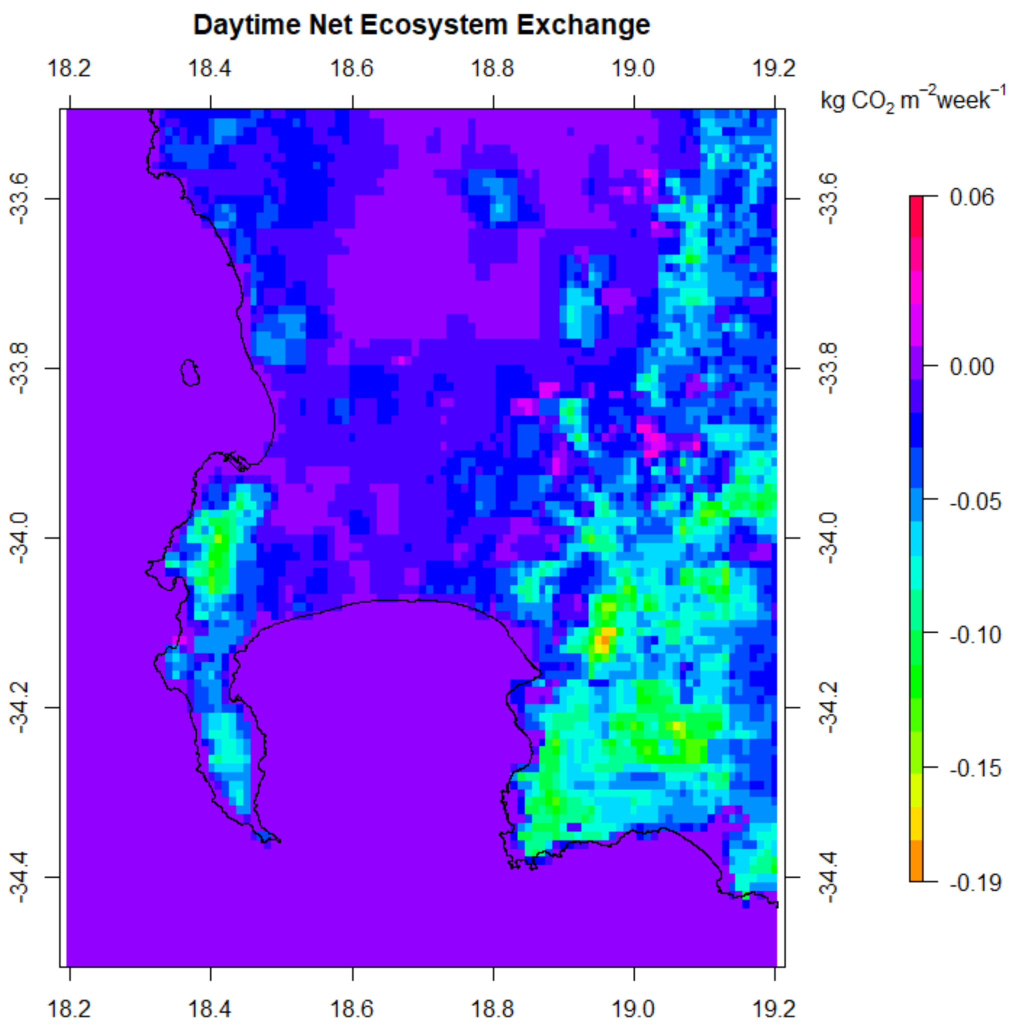


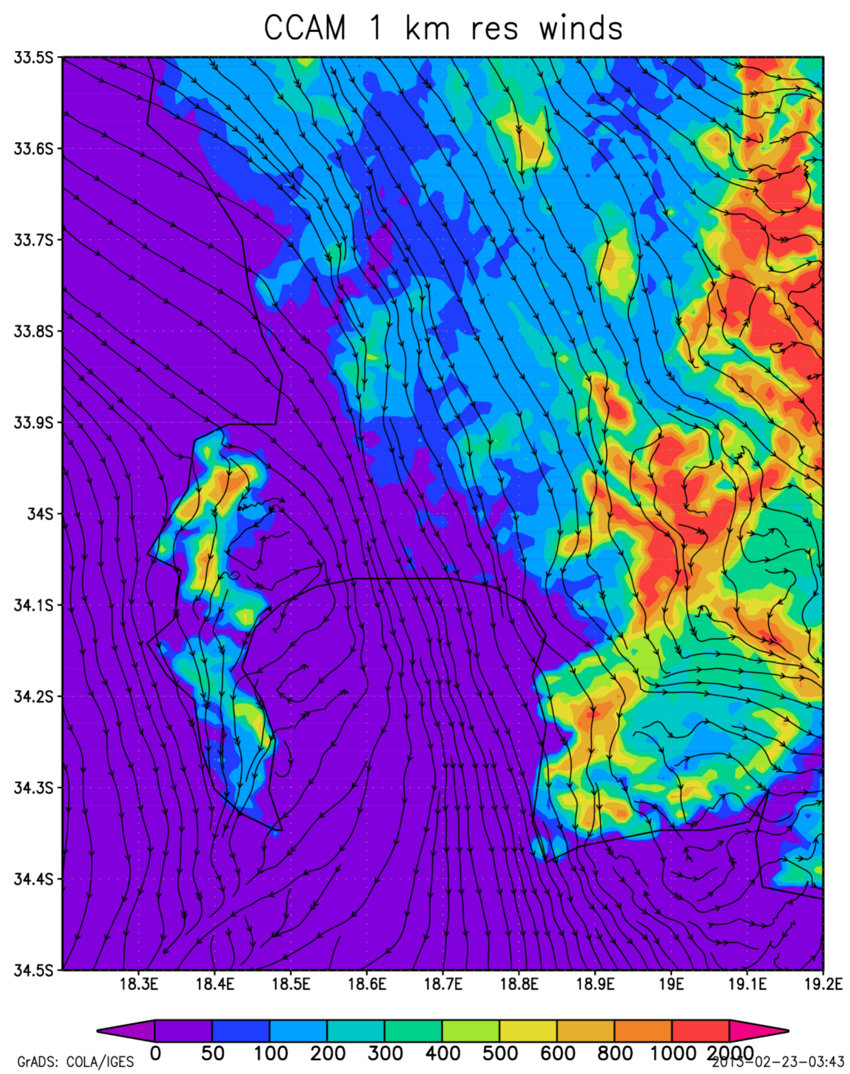


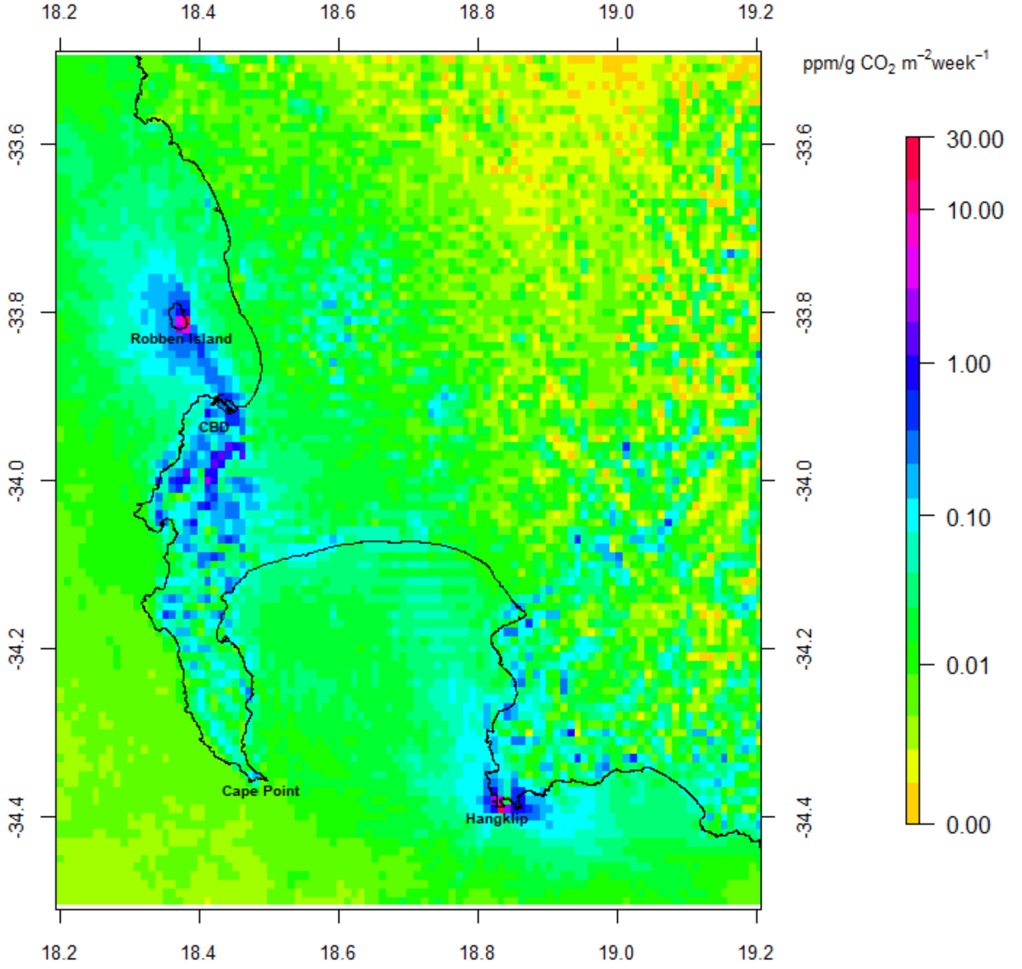


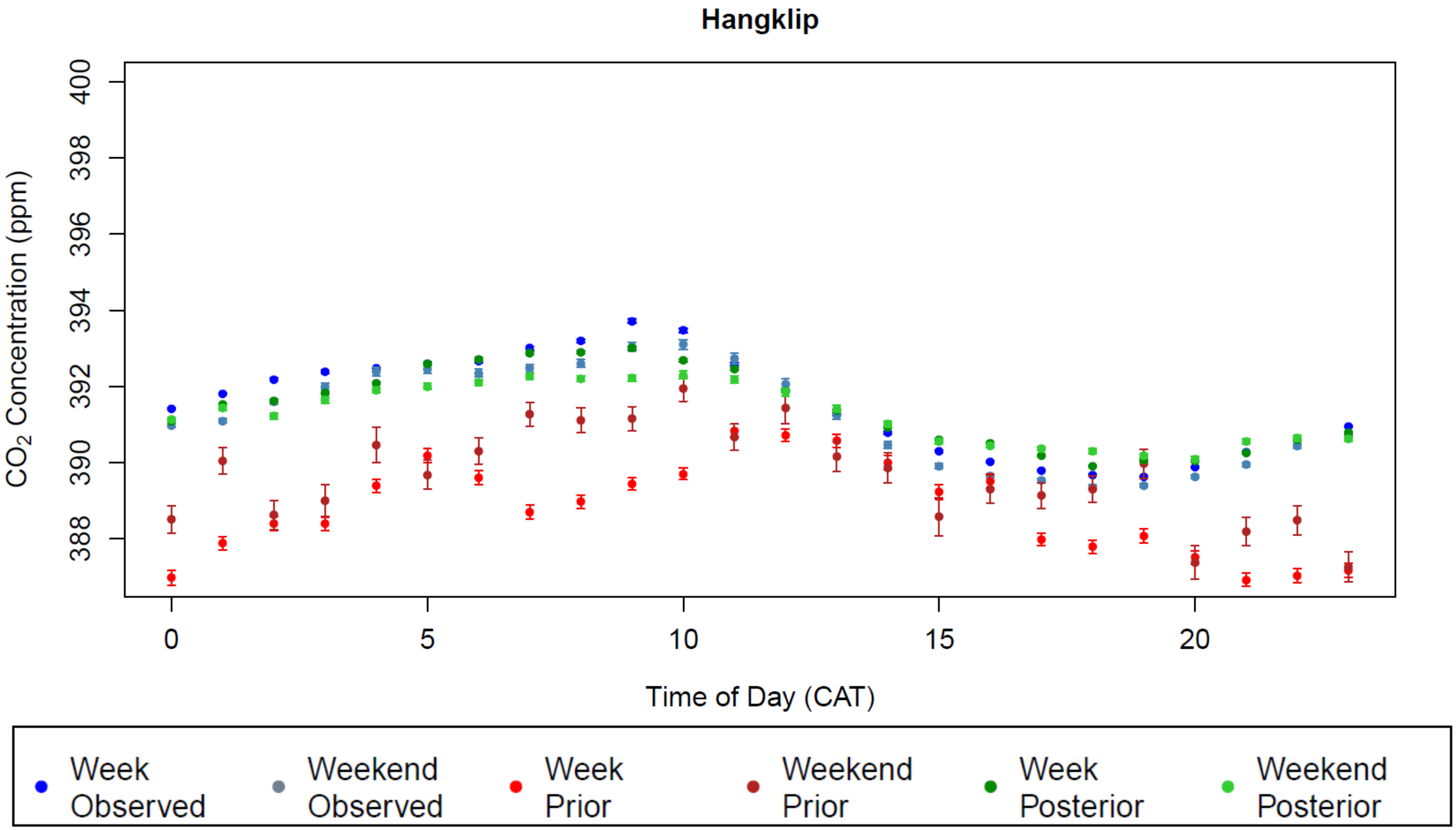


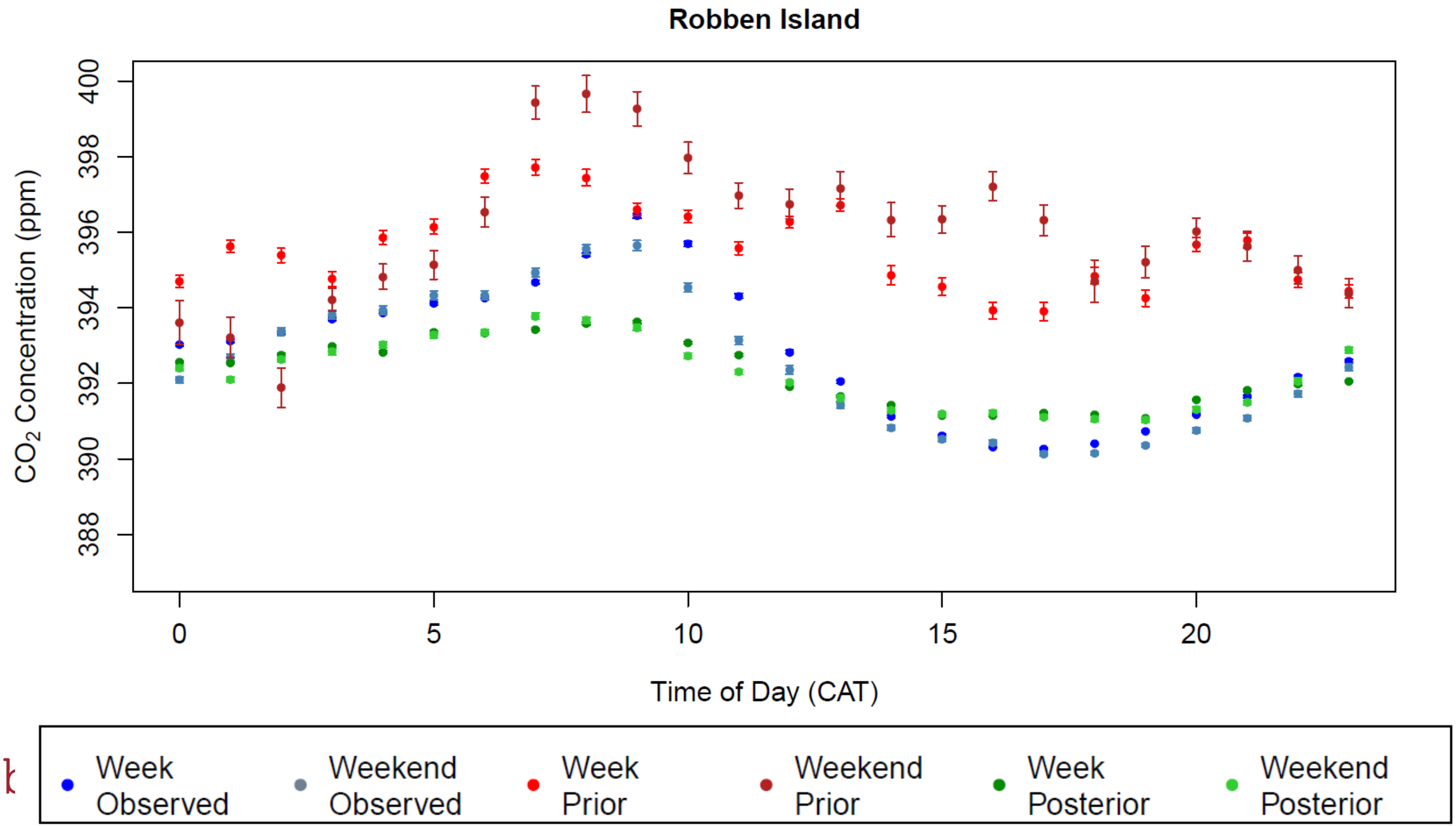
Nickless et al.: Spatial and temporal disaggregation of anthropogenic CO_2 emissions from the City of Cape Town, S. Afr. J. Sci., 111(11/12), Art. #2014-0387, 8 pages, 2015.


Daytime Net Ecosystem Exchange
18.2
18.4
18.6
18.8
19.0
19.2
-33.6
-33.8
-34.0
-34.2
-34.4
kg CO_2 m^{-2} $week^{-1}$
0.06
0.00
-0.05
-0.10
-0.15
-0.19

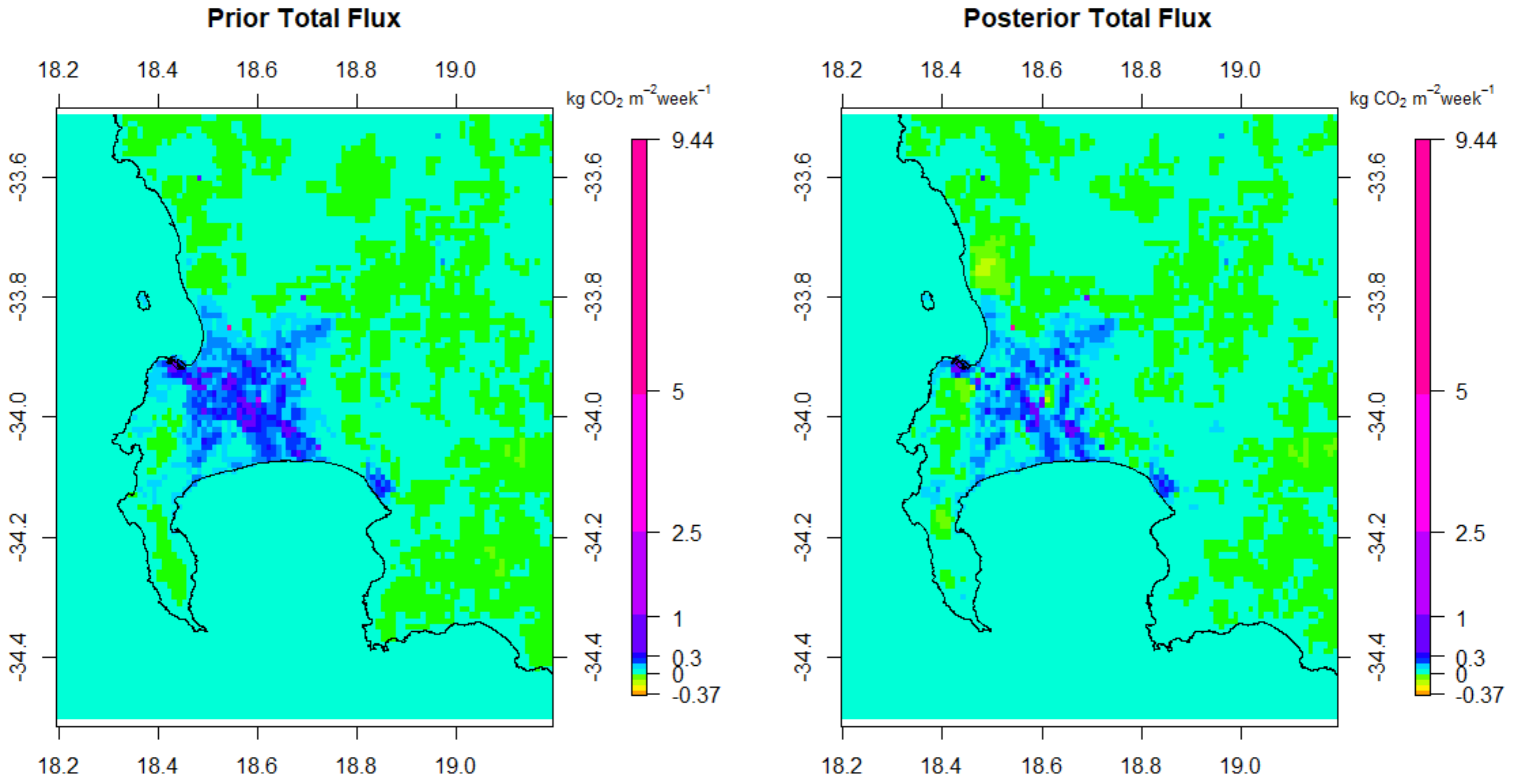

CCAM 1 km res winds
33.5S
33.6S
33.7S
33.8S
33.9S
34S
34.1S
34.2S
34.3S
34.4S
34.5S
18.3E
18.4E
18.5E
18.6E
18.7E
18.8E
18.9E
19E
19.1E
19.2E
0
50
100
200
300
400
500
600
800
1000
2000
GrADS: COLA/IGES
2013-02-23-03:43


18.2
18.4
18.6
18.8
19.0
19.2
-33.6
-33.8
-34.0
-34.2
-34.4
Robben Island
CBD
Cape Point
Hangklip
ppm/g CO_2 $m^{-2}week^{-1}$
30.00
10.00
1.00
0.10
0.01
0.00


Hangklip
CO_2 Concentration (ppm)
388
390
392
394
396
398
400
0
5
10
15
20
Time of Day (CAT)
Week Observed
Weekend Observed
Week Prior
Weekend Prior
Week Posterior
Weekend Posterior

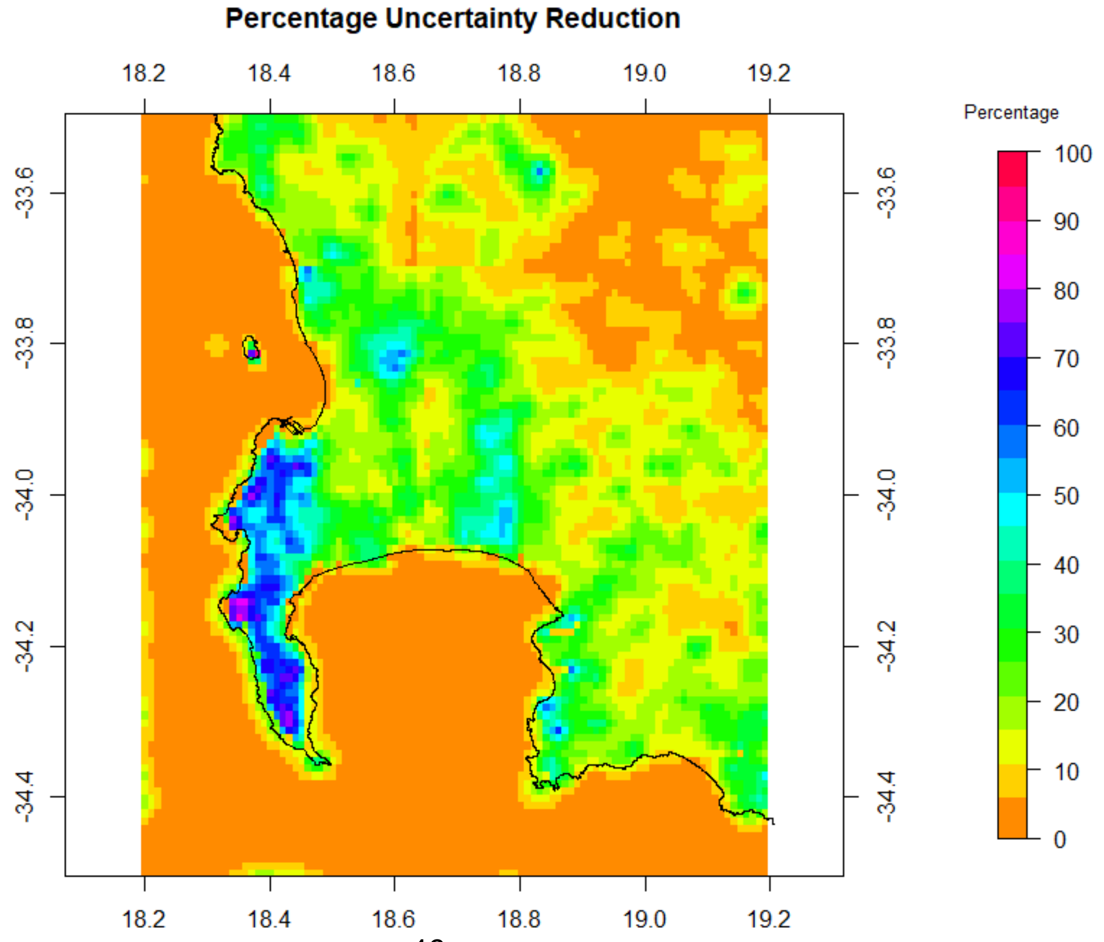

Robben Island
CO$_2$ Concentration (ppm)
388
390
392
394
396
398
400
0
5
10
15
20
Time of Day (CAT)
Week Observed
Weekend Observed
Week Prior
Weekend Prior
Week Posterior
Weekend Posterior

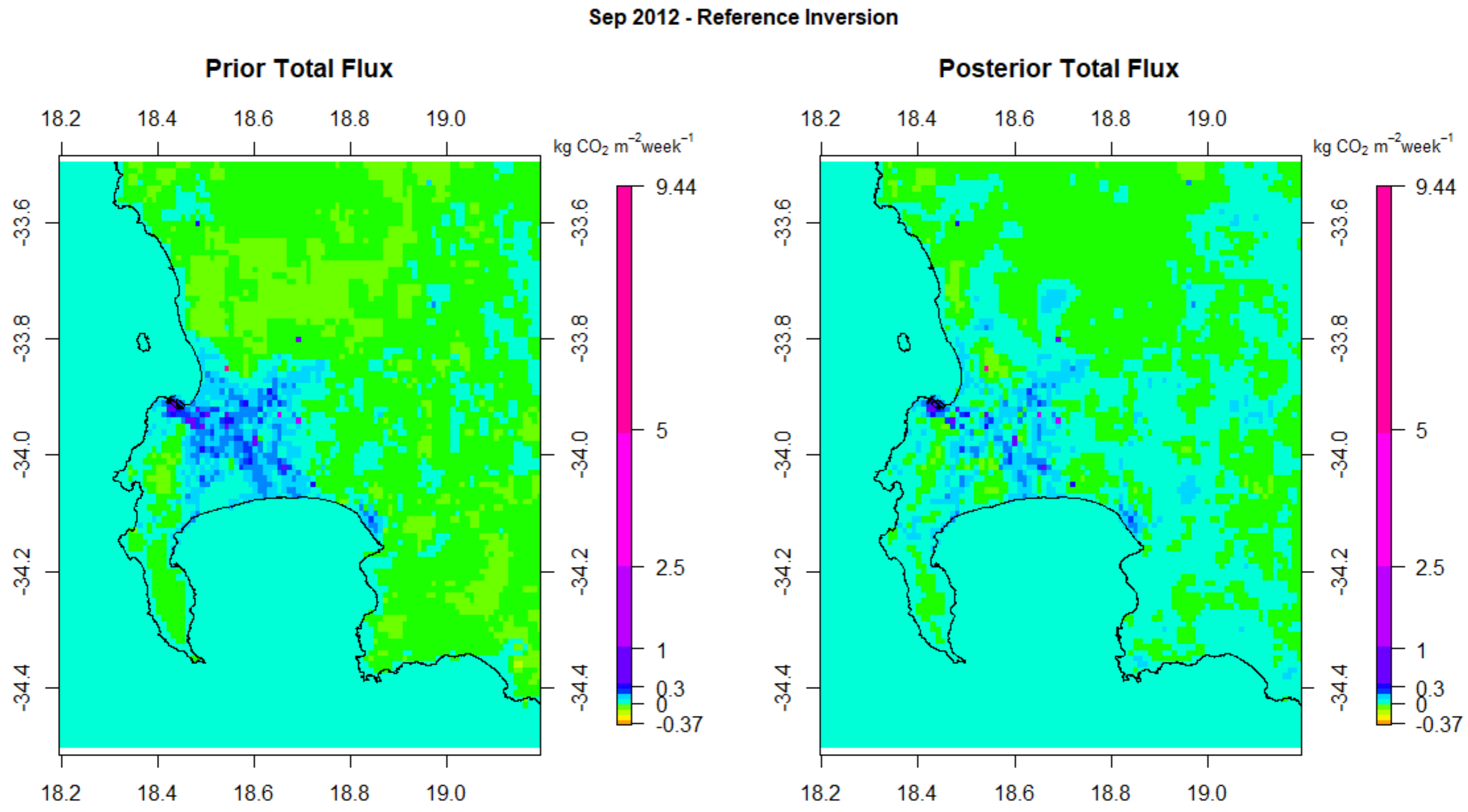
May 2012 - Reference Inversion



Nickless, A., Rayner, P. J., Engelbrecht, F., Brunke, E.-G., Erni, B., and Scholes, R. J.: Estimates of CO2 fluxes over the city of Cape Town, South Africa, through Bayesian inverse modelling, Atmos. Chem. Phys., 18, 4765-4801, https://doi.org/10.5194/acp-18-4765-2018, 2018.

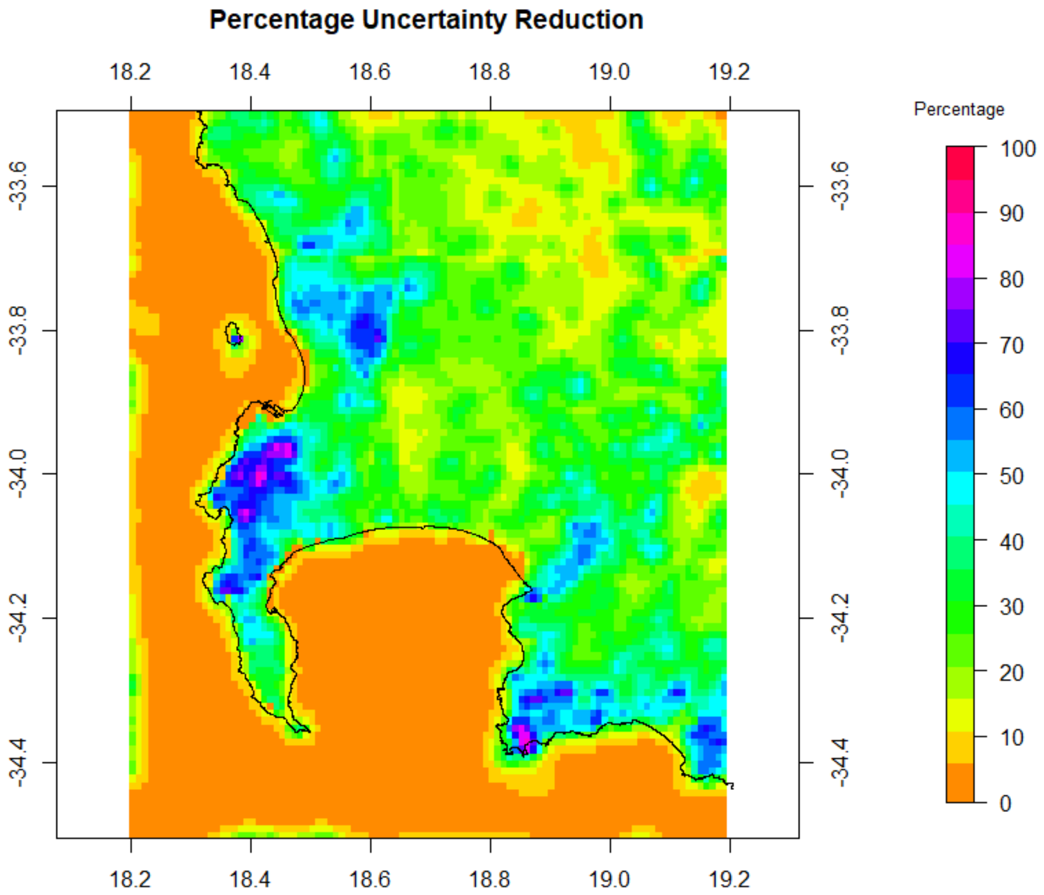
May 2012 - Reference Inversion

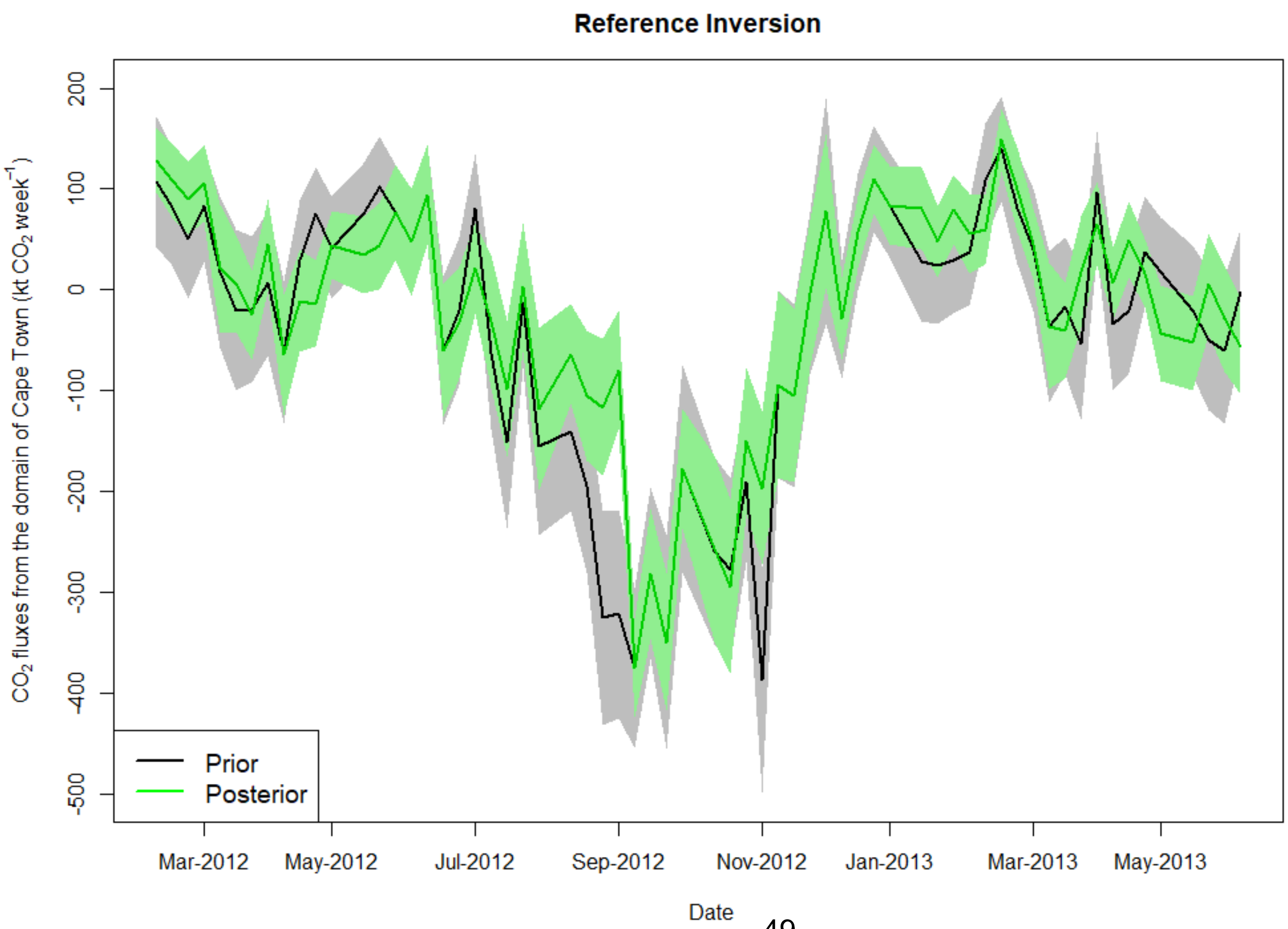




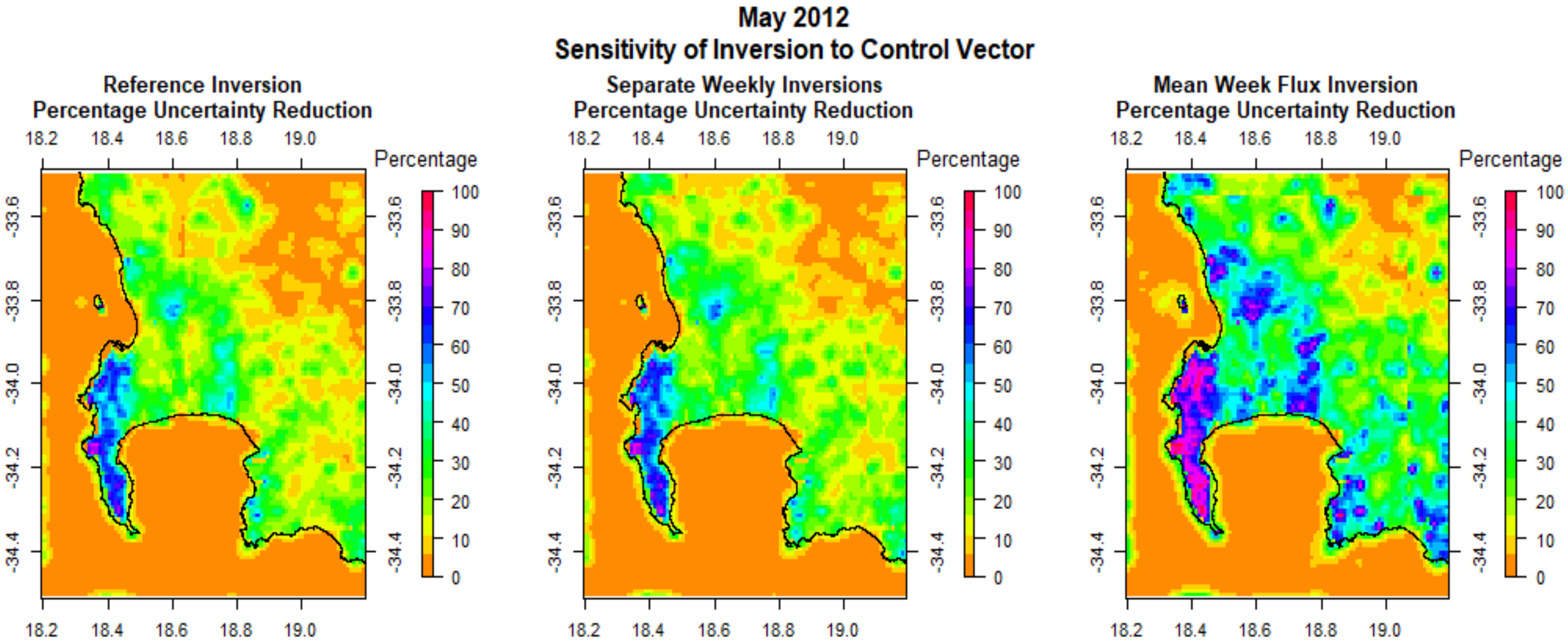
Sep 2012 - Reference Inversion
Prior Total Flux
Posterior Total Flux
18.2
18.4
18.6
18.8
19.0
-33.6
-33.8
-34.0
-34.2
-34.4
kg CO_2 m^{-2} $week^{-1}$
9.44
5
2.5
1
0.3
0
-0.37

September 2012 - Reference Inversion




Reference Inversion
CO_2 fluxes from the domain of Cape Town (kt CO_2 week^{-1})
200
100
0
-100
-200
-300
-400
-500
Prior
Posterior
Mar-2012
May-2012
Jul-2012
Sep-2012
Nov-2012
Jan-2013
Mar-2013
May-2013
Date

Sensitivity analysis – control vector



Nickless, A., Rayner, P. J., Scholes, R. J., Engelbrecht, F., and Erni, B.: An atmospheric inversion over the city of Cape Town: sensitivity analyses, Atmos. Chem. Phys. Discuss., doi: 10.5194/acp-2018-535, in review, 2018.

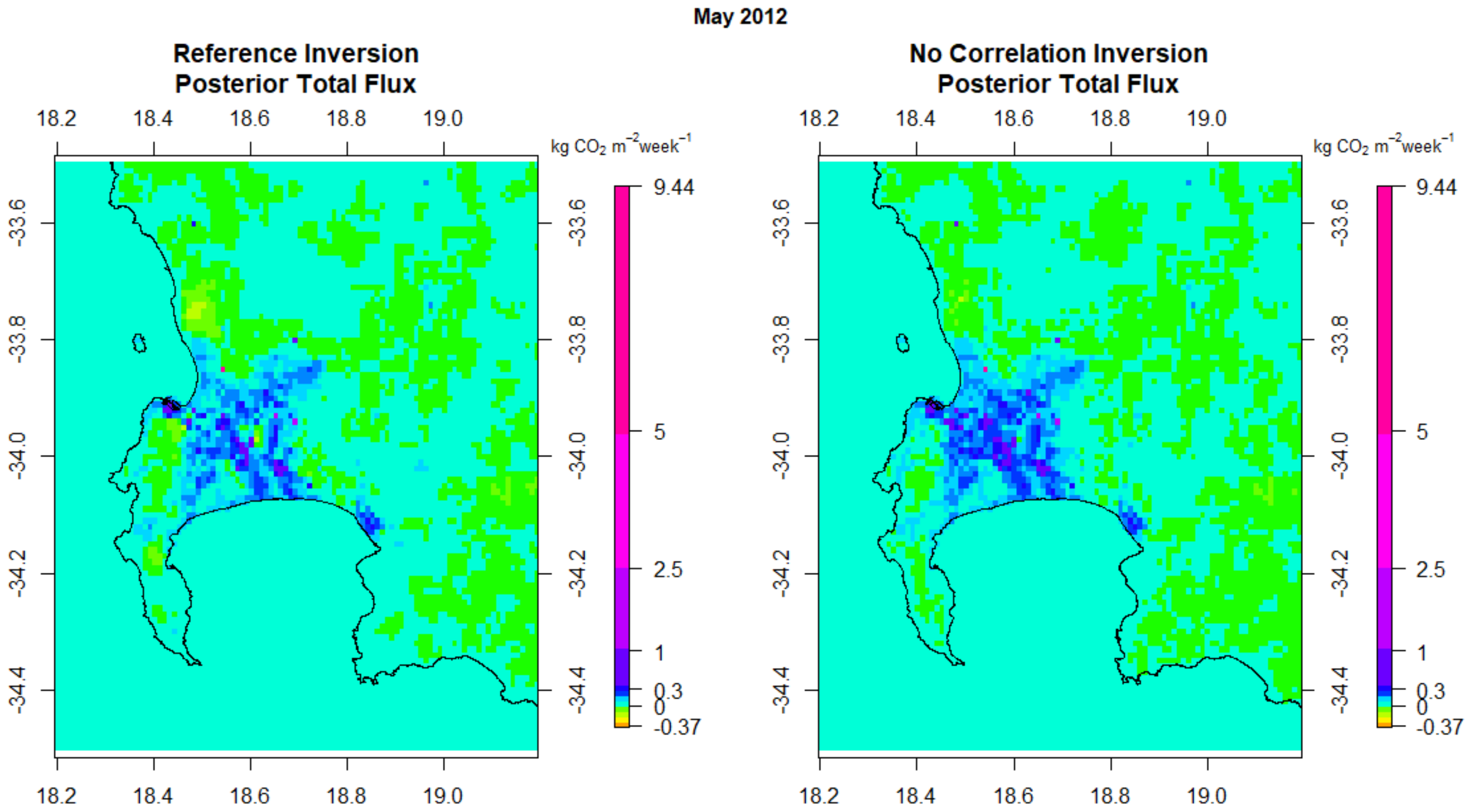
Error correlation

The off-diagonal elements of $\mathbf{C_c}$ were calculated, based on the Balgovind correlation model as used in Wu et al. (2013)

$$C_{s_0;NEE}\left(s_{NEE;i}, s_{NEE;j}\right) = \sqrt{C_{s_0;NEE}\left(s_{NEE;i}\right)}\sqrt{C_{s_0;NEE}\left(s_{NEE;j}\right)}\left(1+\frac{h}{L}\right)\exp\left(-\frac{h}{L}\right)$$

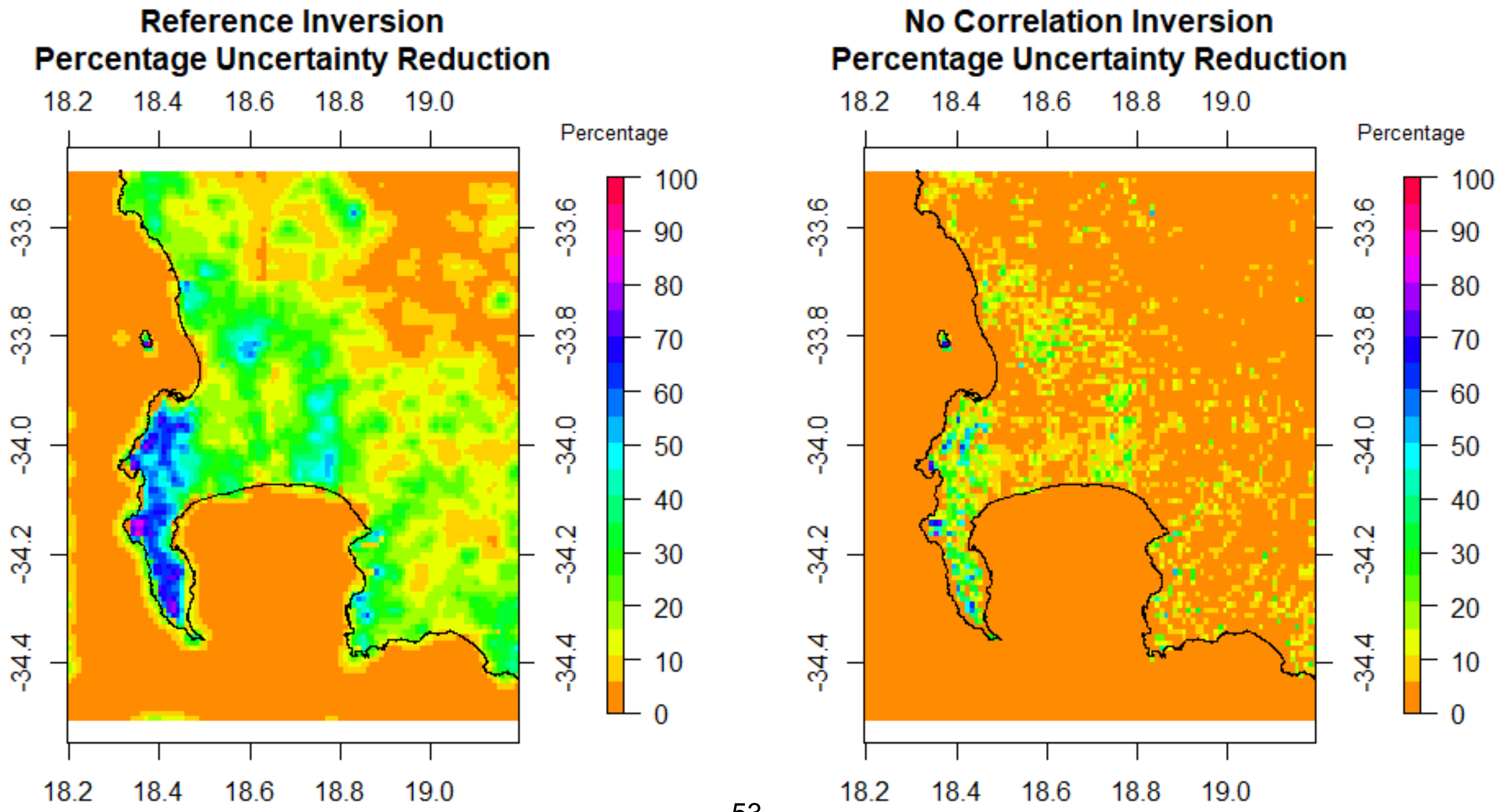
where $s_{NEE;i}$ and $s_{NEE;j}$ are NEE fluxes in pixels i and j, $C_{s_0;NEE}\left(s_{NEE;i}\right)$ and $C_{s_0;NEE}\left(s_{NEE;j}\right)$ the corresponding variances in the NEE flux uncertainties in pixels i and j, the characteristic correlation length L was assumed to be 1 km, and h is the spatial distance pixels i and j.

Sensitivity analysis – error correlations



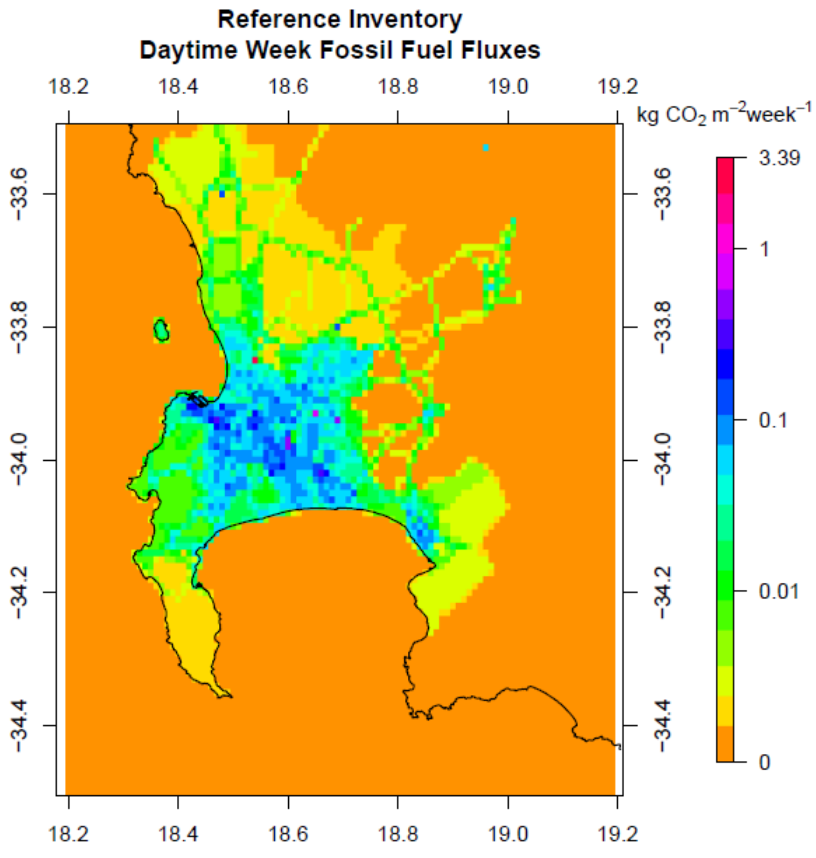
b:

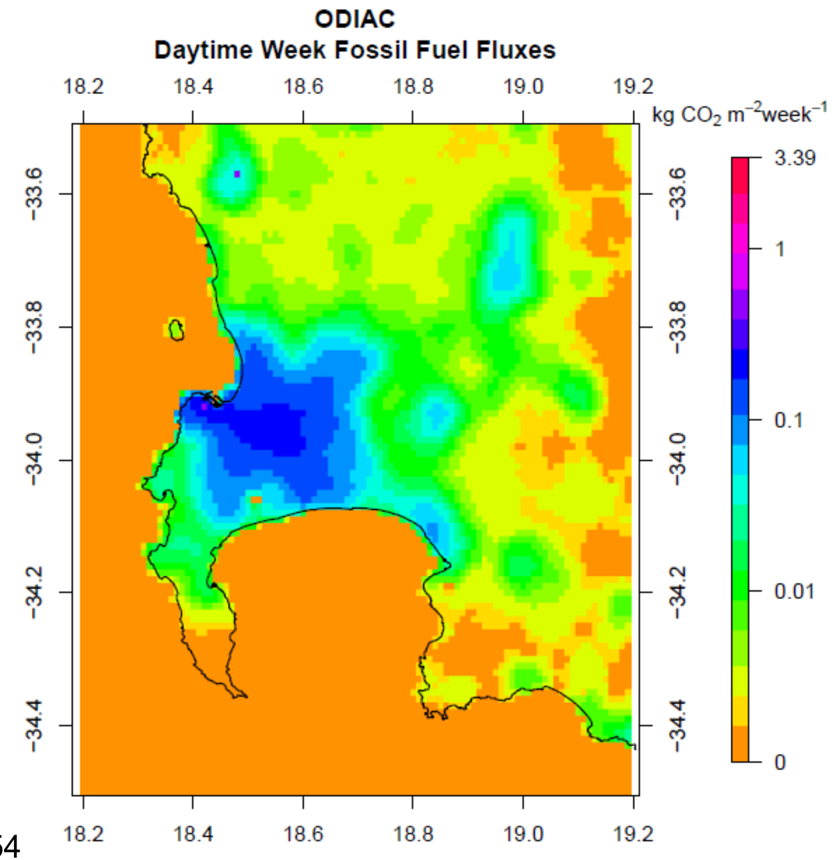
May 2012

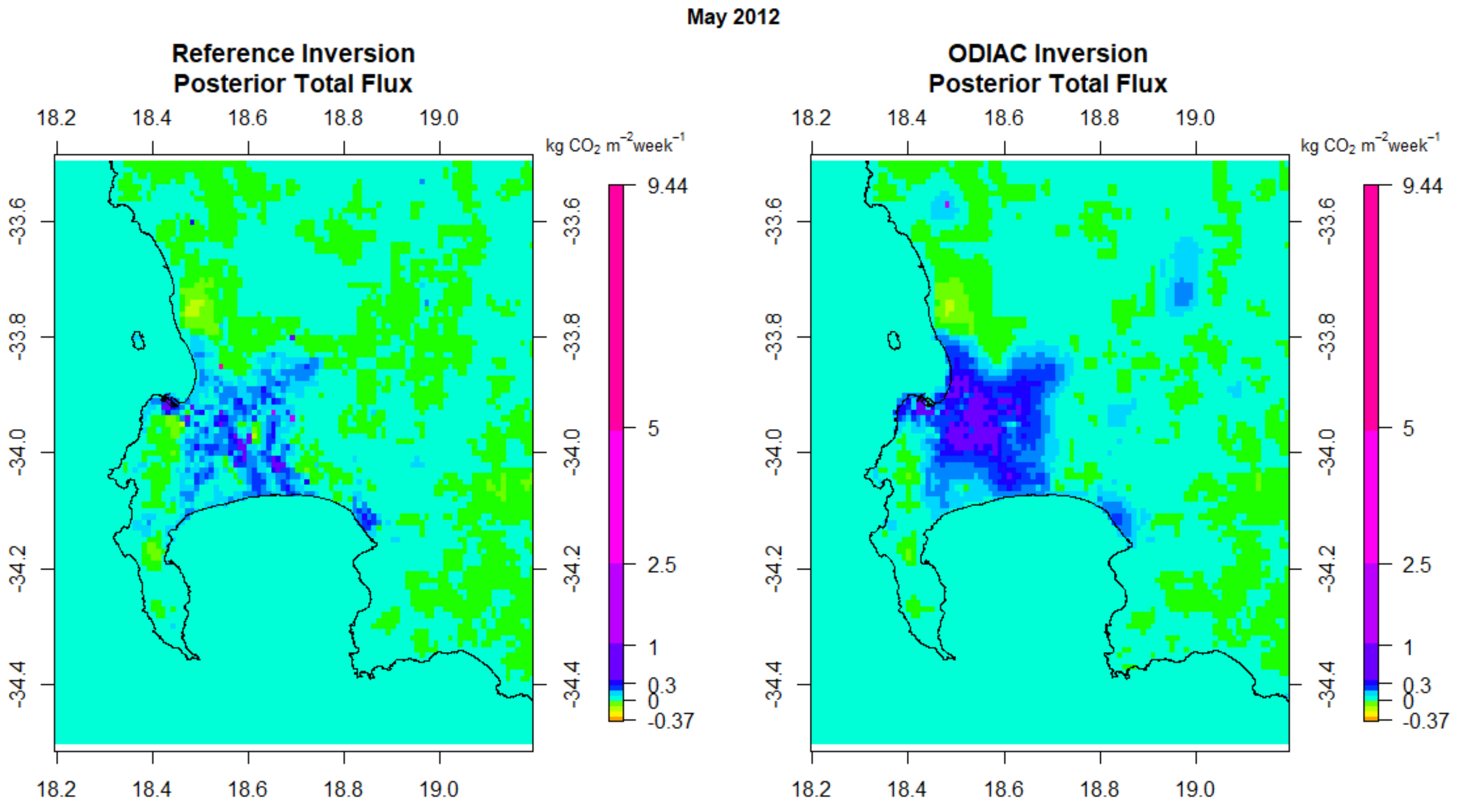


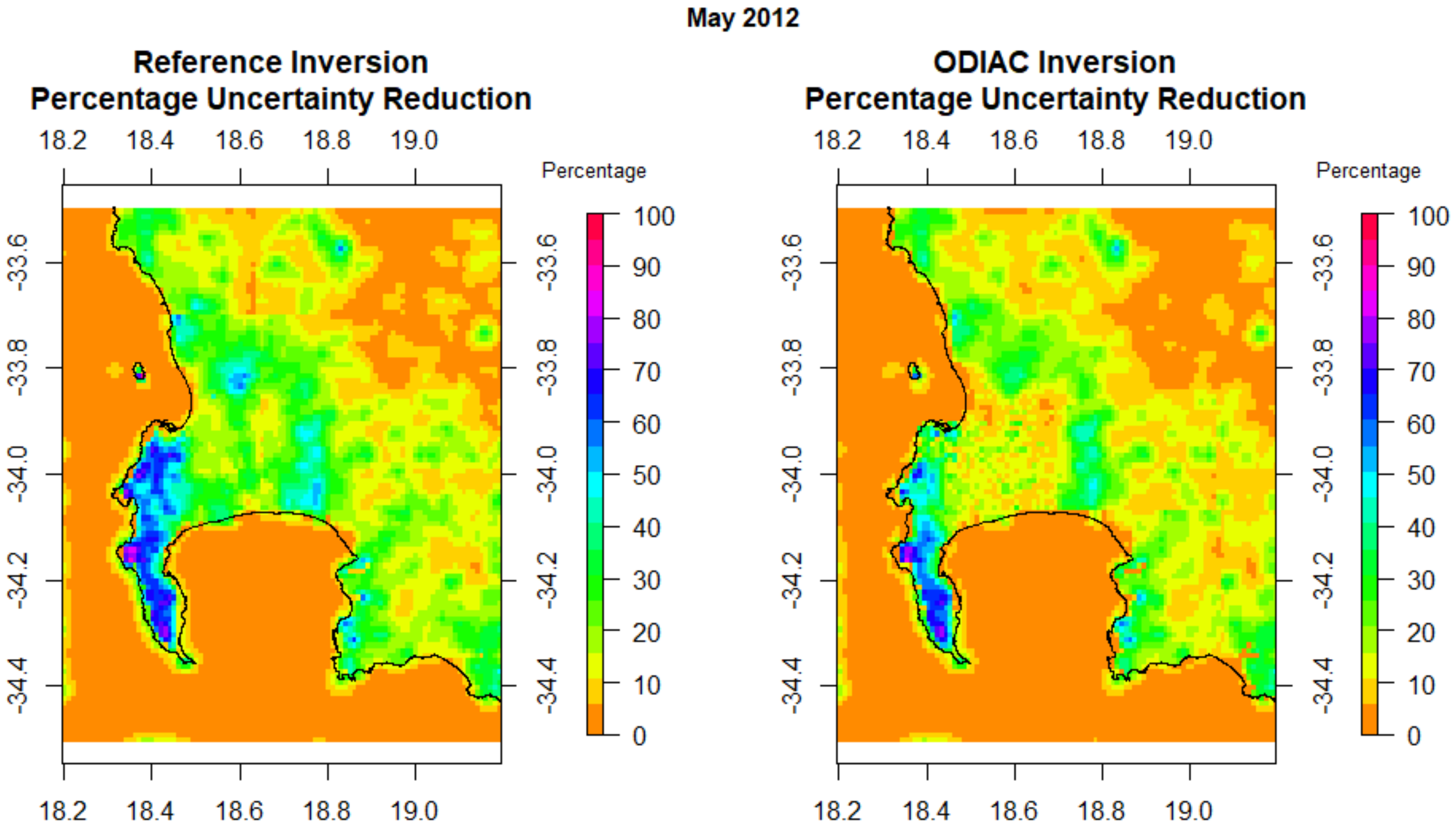
Reference Inversion
Percentage Uncertainty Reduction
No Correlation Inversion
Percentage Uncertainty Reduction
Percentage
18.2
18.4
18.6
18.8
19.0
-33.6
-33.8
-34.0
-34.2
-34.4
0
10
20
30
40
50
60
70
80
90
100

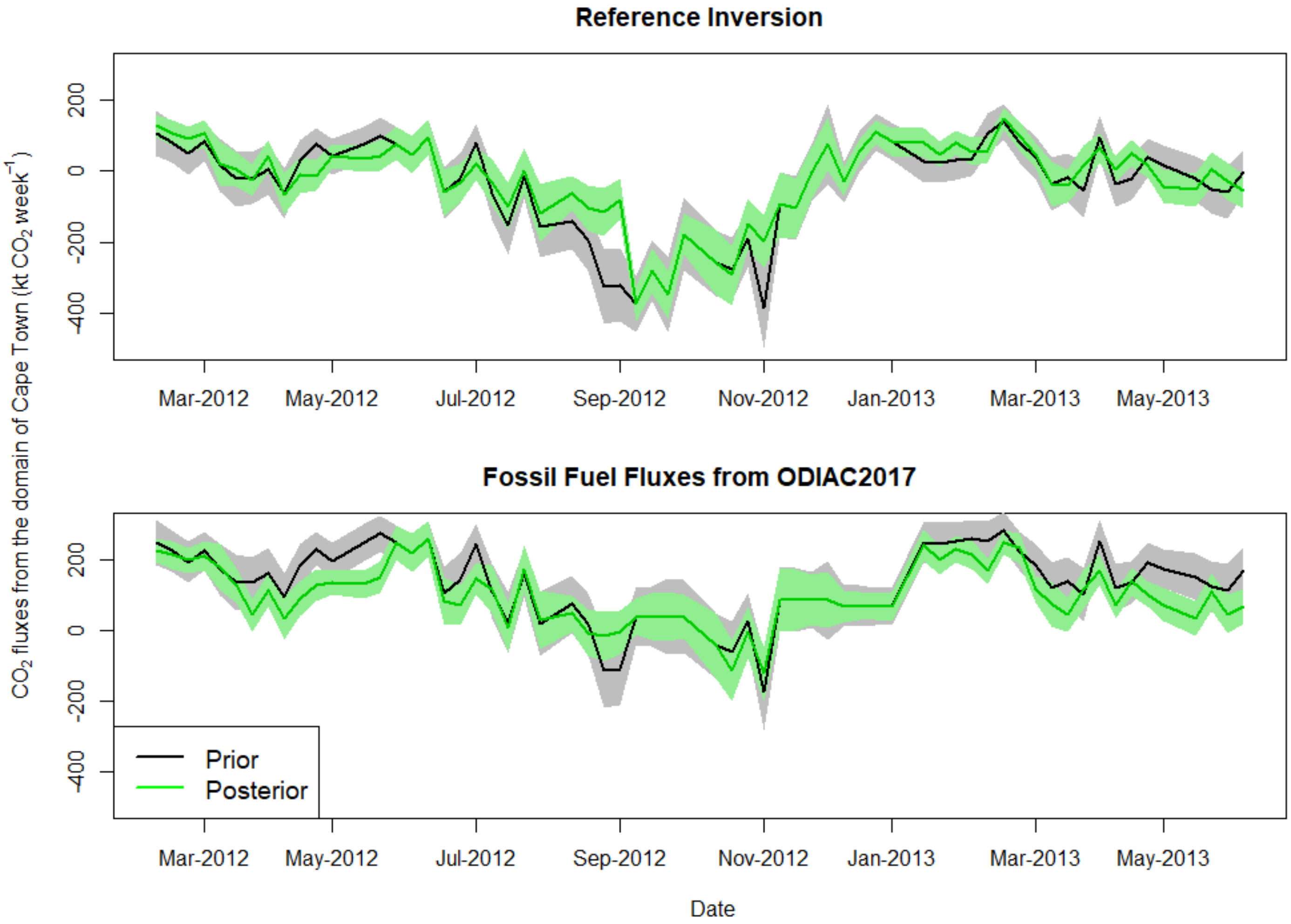
Sensitivity analysis – prior information

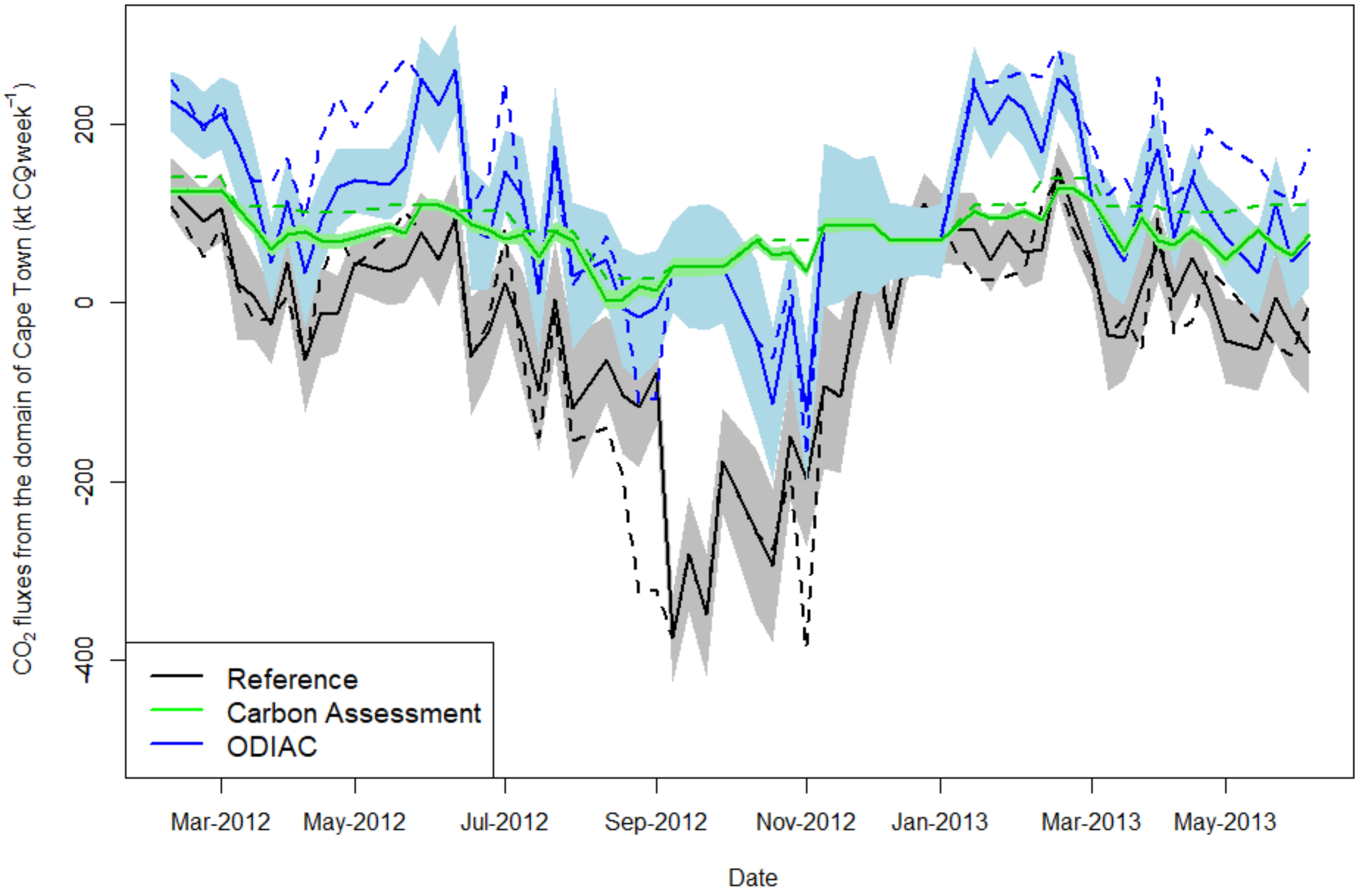





May 2012
Reference Inversion
Posterior Total Flux
ODIAC Inversion
Posterior Total Flux
kg CO2 m-2week-1
9.44
5
2.5
1
0.3
0
-0.37
18.2
18.4
18.6
18.8
19.0
-33.6
-33.8
-34.0
-34.2
-34.4

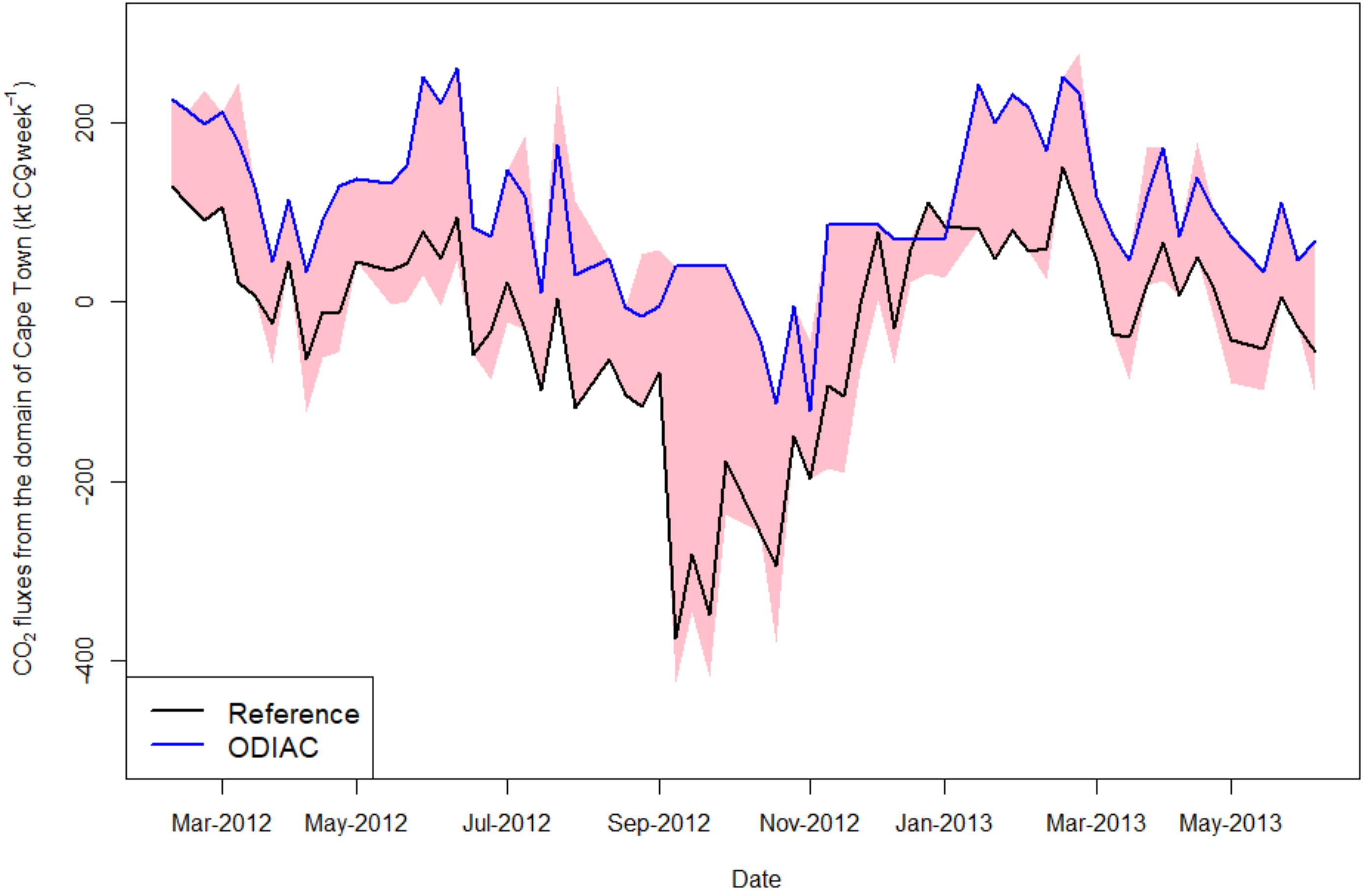

May 2012
Reference Inversion
Percentage Uncertainty Reduction
ODIAC Inversion
Percentage Uncertainty Reduction
Percentage
18.2
18.4
18.6
18.8
19.0
-33.6
-33.8
-34.0
-34.2
-34.4
0
10
20
30
40
50
60
70
80
90
100


Reference Inversion
Fossil Fuel Fluxes from ODIAC2017
CO_2 fluxes from the domain of Cape Town (kt CO_2 week^{-1})
200
0
-200
-400
Mar-2012
May-2012
Jul-2012
Sep-2012
Nov-2012
Jan-2013
Mar-2013
May-2013
Date
Prior
Posterior



CO2 fluxes from the domain of Cape Town (kt CO2 week-1)
200
0
-200
-400
Mar-2012
May-2012
Jul-2012
Sep-2012
Nov-2012
Jan-2013
Mar-2013
May-2013
Date
Reference
Carbon Assessment
ODIAC

Combining information from inversions with different prior products



Model Assessment

$$\chi_1^2 = \frac{1}{n}(\mathbf{Hs_0} - \mathbf{c})^T\left(\mathbf{HC}_{s_0}\mathbf{H}^T + \mathbf{C_c}\right)^{-\mathbf{1}}(\mathbf{Hs_0} - \mathbf{c})$$

Where n is the dimension of the data space.

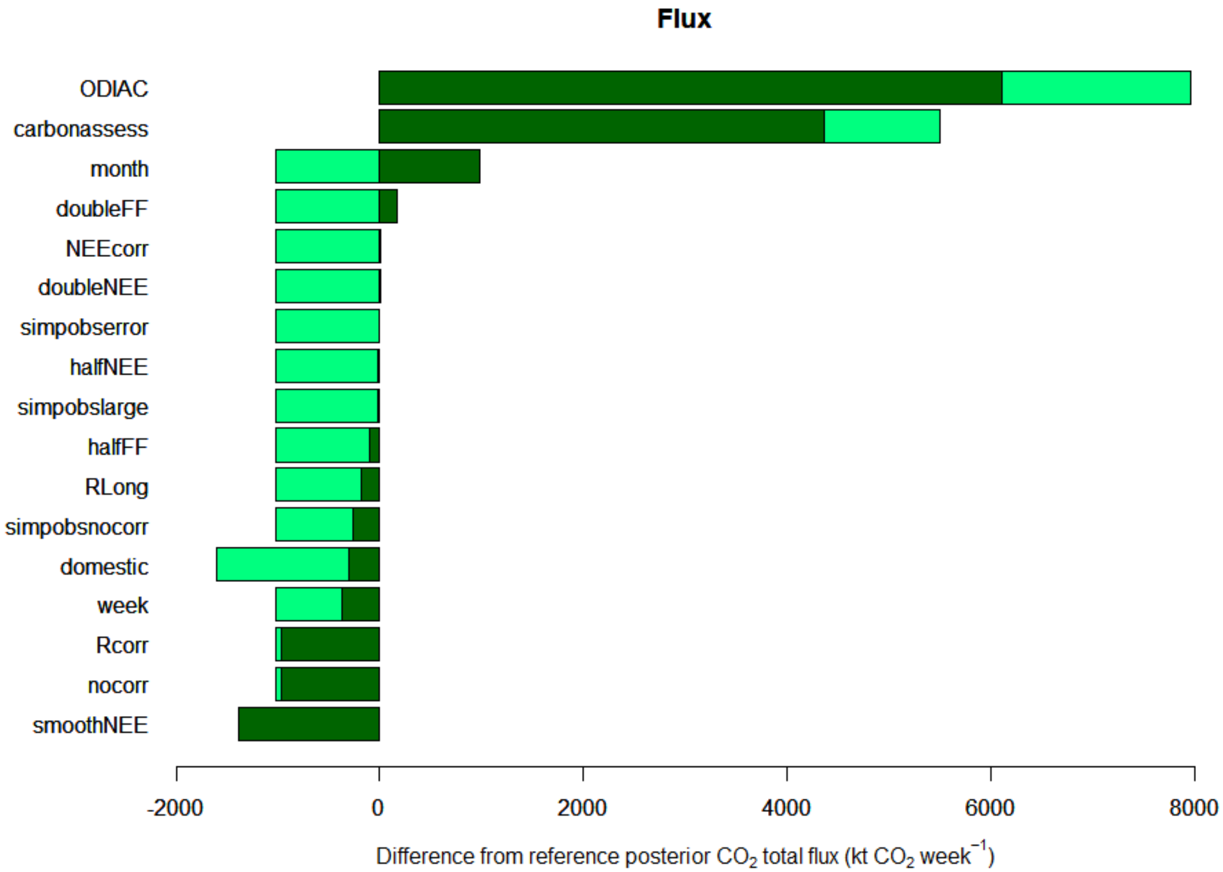
The squared residuals from the inversion (squared differences between observed and modelled concentrations) should follow the χ^2 distribution with degrees of freedom equal to the number of observations (Michalak et al., 2005; Tarantola, 2005). Dividing this statistic by the degrees of freedom should yield a χ_1^2 distribution. Values lower than one indicate that the uncertainty is too large, and values greater than one indicate that the uncertainty prescribed is lower than it should be. The error in the assignment of the uncertainty could be in either $\mathbf{C_c}$ or $\mathbf{C}_{s_0}$ (or both).

	Reference	Carbon Assessment	ODIAC	NEE Correlation Only	Obs Error Correlation Only	No Correlation
Prior Flux (sd)	-1336 (254)	5181 (32)	7635 (256)	-1336 (254)	-1336 (63)	-1336 (63)
Posterior Flux (sd)	-317 (189)	4045 (28)	5787 (195)	-310 (189)	-1281 (59)	-1287 (59)
Uncertainty Reduction	25.6%	11.9%	23.6%	25.6%	7.5%	7.5%
Mean Chi-Squared Statistic	1.48 (0.55)	4.13 (1.24)	1.25 (0.49)	1.49 (0.54)	2.1 (0.78)	2.12 (0.79)

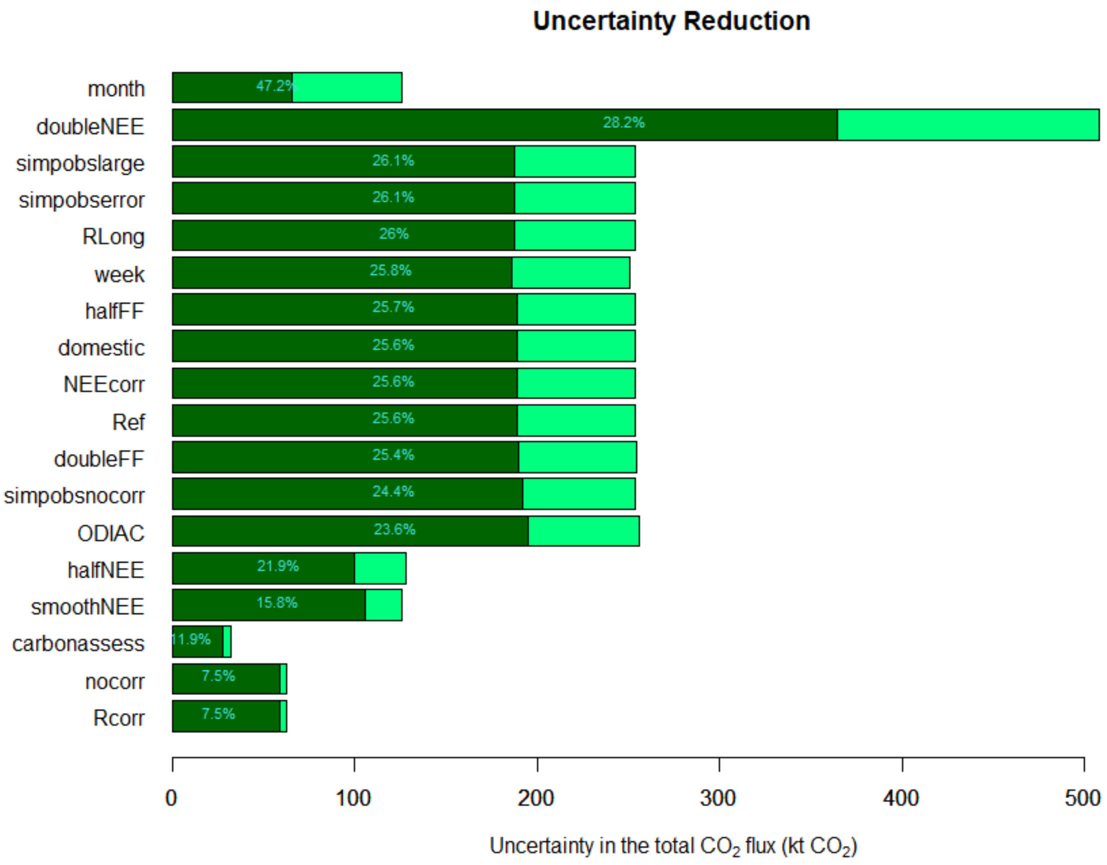
	Double Fossil Fuel Uncertainty	Half Fossil Fuel Uncertainty	Double NEE Uncertainty	Half NEE Uncertainty	Domestic Emissions Smoothed	NEE Smoothed
Prior Flux (sd)	-1336 (255)	-1336 (254)	-1336 (508)	-1336 (128)	-1916 (254)	-1328 (126)
Posterior Flux (sd)	-151 (190)	-423 (189)	-316 (365)	-337 (100)	-624 (189)	-1707 (106)
Uncertainty Reduction	25.4%	25.7%	28.2%	21.9%	25.6%	15.8%
Mean Chi-Squared Statistic	1.21 (0.5)	1.86 (0.63)	1.03 (0.47)	2.22 (0.69)	1.41 (0.49)	1.17 (0.47)

	Simple Obs Error	Simple Obs Error with Large Night	Simple Obs Error No Correlation	Obs Error Correlation Long	Mean Monthly Inversion	Weekly Inversions
Prior Flux (sd)	-1336 (254)	-1336 (254)	-1336 (254)	-1336 (254)	-1336 (126)	-1220 (251)
Posterior Flux (sd)	-325 (188)	-338 (188)	-579 (192)	-497 (188)	662 (66)	-687 (186)
Uncertainty Reduction	26.1%	26.1%	24.4%	26.0%	47.2%	25.8%
Mean Chi-Squared Statistic	2.17 (1.04)	1.88 (0.92)	2.25 (1.13)	7.3 (6.2)	1.43 (0.55)	1.54 (0.56)

Sensitivity Analysis – Size of Posterior Flux



Sensitivity Analysis – Uncertainty Reduction



An optimal observation network for Africa – using an inverse modelling approach

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 730995

Uncertainty reduction

- Solution for the posterior uncertainty covariance matrix does not require the observed concentrations from a site.

$$\mathbf{C_s} = \mathbf{C}_{\mathbf{s}_0} - \mathbf{C}_{\mathbf{s}_0}\mathbf{H}^T\left(\mathbf{H}\mathbf{C}_{\mathbf{s}_0}\mathbf{H}^T + \mathbf{C_c}\right)^{-1}\mathbf{H}\mathbf{C}_{\mathbf{s}_0}$$

- Can compare to prior uncertainty to assess the uncertainty reduction a set of new measurement sites will contribute.

$$J_{Ce} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m} C_{s_{ij}}}$$

$$\text{Uncertainty reduction} = 1 - \frac{\widehat{J_{Ce}}}{J_{Ce\,base}}$$

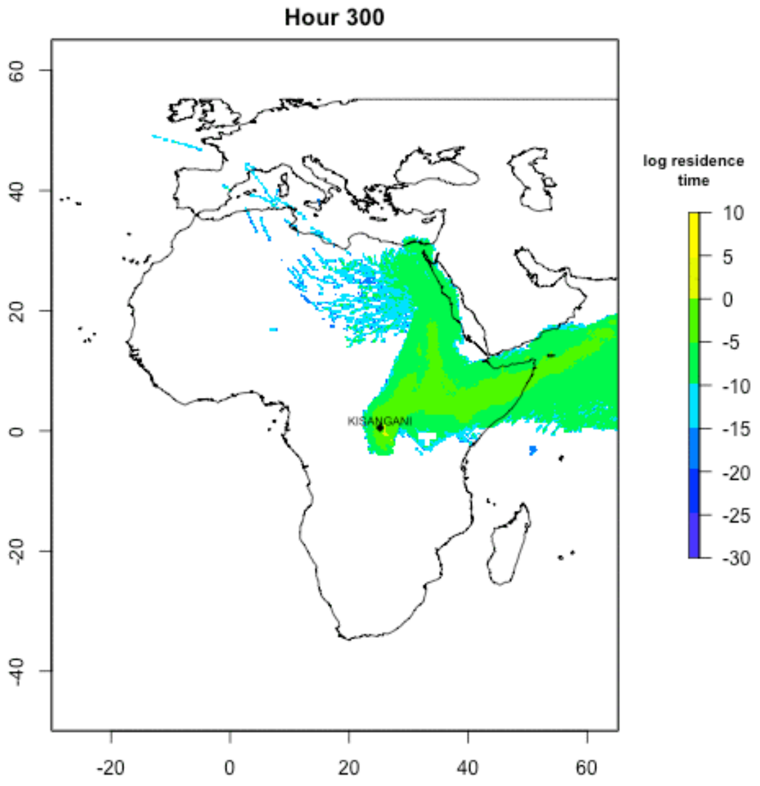
Incremental Optimisation

- Solves for one site at a time.
- Starts off by finding the first site to add to the existing network that reduces the uncertainty by the most.
- Keeps adding the best of the remaining sites to the network until the required uncertainty reduction or network size is reached.
- Works well for observation network designs.

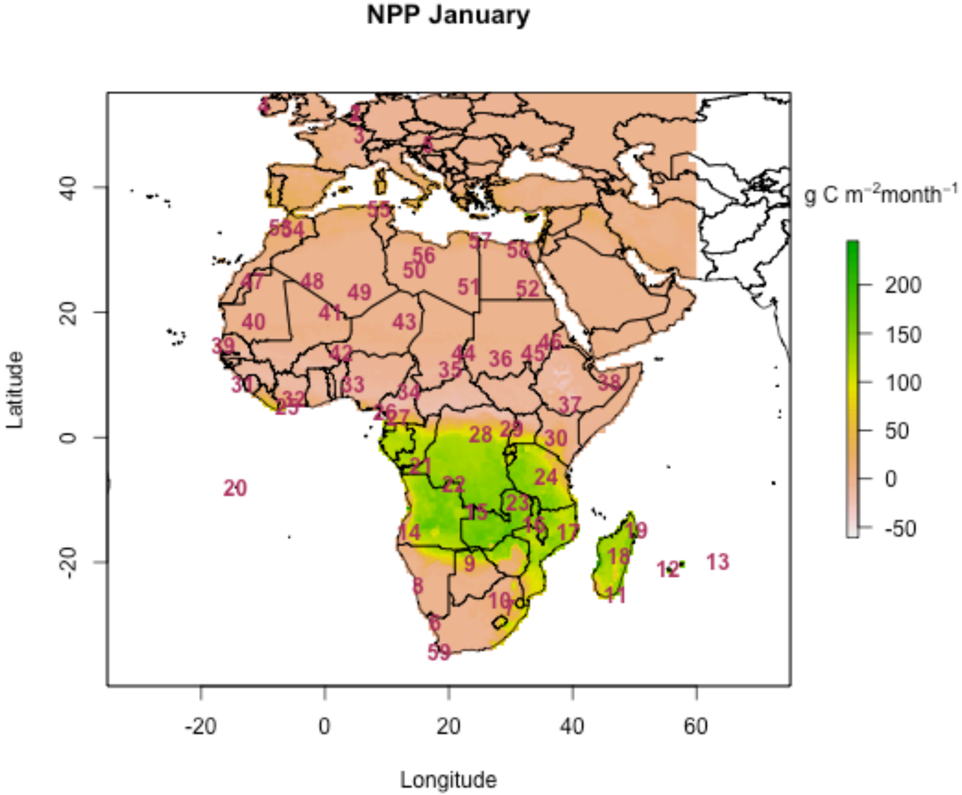
Transport Model

- FLEXPART version 10.3 – Lagrangian Particle Dispersion Model
- The meteorology driving the LPDM is generated by the European Centre for Medium-Range Weather Forecasts ERA-Interim meteorological analyses.
- Releases parcels of air from the receptor sites and runs backward in time to determine sensitivity to each source.
- 60 000 released every 3 hours, and parcels remain live for 10 days.
- The sensitivity, which is expressed in residence time, converts a flux into a mass concentration.

Sensitivities

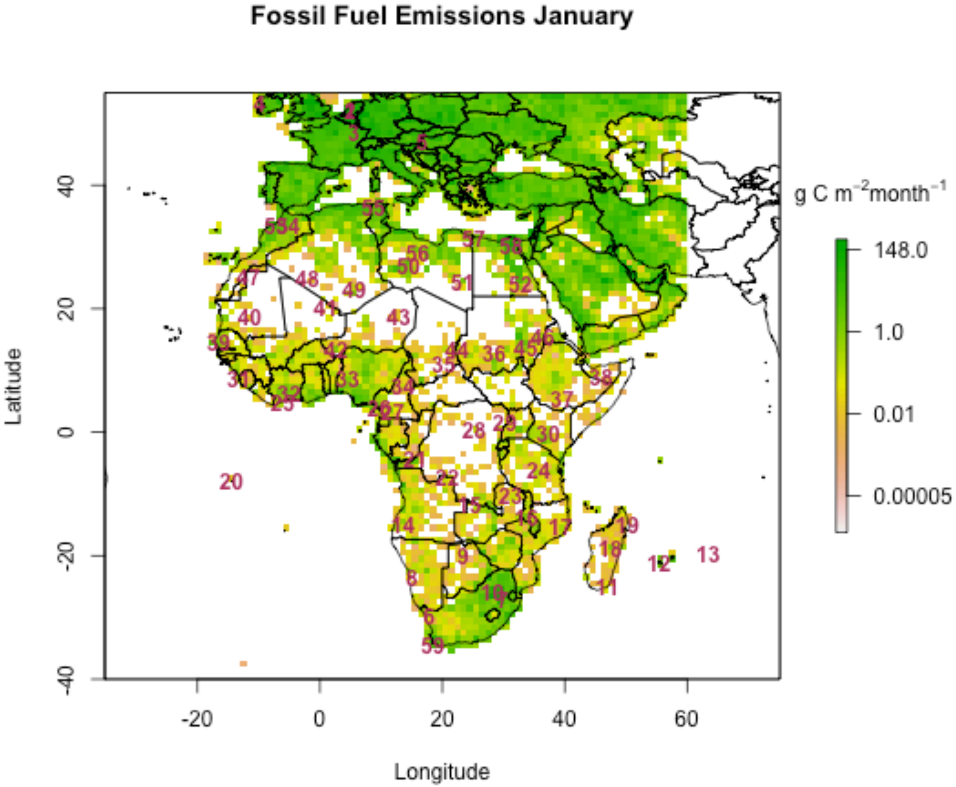


Prior Biogenic CO_2 Flux Uncertainties



bristol.ac.uk

Prior CO_2 Fossil Fuel Flux Uncertainties



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